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The Geophysical Journal

OF THE

ROYAL ASTRONOMICAL SOCIETY

Editors

A. H. Cook
M.A., Ph.D., F.R.A.S., F.G.S.
National Physical Laboratory
Teddington

T. F. Gaskell
M.A., Ph.D., F.R.A.S.
British Petroleum Company
London

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The Assistant Secretary

Royal Astronomical Society, Burlington House, London, W.1.

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MEETING OF 1959 JANUARY 9

Dr W. H. Steavenson, President, in the Chair

The President announced the death of William Otto Brunner, an Associate of the Society, and paid a brief tribute to his memory, the Fellows standing.

The President announced that the Council had awarded the Gold Medal of the Society to Dr Raymond Arthur Lyttleton for his outstanding contributions to many branches of theoretical astronomy, in particular to the problems of the stability of rotating fluid masses, of the constitution of the stars and stellar evolution, and of the origin and peculiarities of the solar system.

The President announced that the Council had awarded the Eddington Medal to Dr James Stanley Hey for his outstanding discoveries concerning radio emission from the active Sun, radio reflections from meteors, and discrete sources of cosmic radio emission.

The election by the Council of the following Fellows was duly confirmed:—

Dennis Ian Gough, University College of Rhodesia and Nyasaland, Private Bag 167H, Salisbury, S. Rhodesia (proposed by A. L. Hales);

Edward Irving, Department of Geophysics, Australian National University, Canberra, Australia (proposed by S. C. B. Gascoigne);

Colin Stewart Lindsay Keay, 24 Cashel Street, Christchurch, New Zealand (proposed by R. F. Joyce);

David Layzer, Harvard College Observatory, Cambridge, Mass., U.S.A. (proposed by T. Gold);

Rex Wilheim Mallett, 40 Queen's Avenue, Warrington, Lancs. (proposed by A. C. Sanderson);

Arthur Jack Meadows, University Observatory, Oxford (proposed by H. H. Plaskett);

Colin Robert Munford, 13 Ladywood Road, Ipswich, Suffolk (proposed by R. G. Andrews);

Alan Eben MacKenzie Nairn, Physics Department, King's College, Newcastle-upon-Tyne, 1 (proposed by S. K. Runcorn); and

*Francis Charles Wykes, 76 Culverden Park Road, Tunbridge Wells, Kent (proposed by C. Hunt).

* Transferring from Junior Membership.

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*Francis Charles Wykes, 76 Culverden Park Road, Tunbridge Wells, Kent (proposed by C. Hunt).

* Transferring from Junior Membership.

The re-election by the Council of the following Fellow was duly confirmed:—

Arthur Francis O'Donel Alexander, 1 Athelstan Road, Dorchester, Dorset (proposed by J. R. Bazin).

The election by the Council of the following Junior Member was duly confirmed:—

Jeremy Benson Tatum, 19 Montpelier Road, London, N.3 (proposed by C. W. Allen).

Eighty-four presents were announced as having been received since the last meeting, including:—

A. Argoli: *Ephemerides* ... 1641–1670 (presented by K. S. Jewson);

J. Gadbury: *Ephemerides* ... 1719–1740 (presented by K. S. Jewson); and

B. Lehnert, (Ed.): *Electromagnetic phenomena in cosmical physics. Symposium No.* 6 *of the I.A.U.* (presented by the I.A.U.).

ON THE SHAPE OF COMETARY TAILS

B. L. Meek

(Communicated by H. Bondi)

(Received 1958 October 24)

Summary

On the basis of a particle swarm structure for a comet, the subsequent space distribution of particles of different sizes produced within it near perihelion is calculated for a series of later instants. The separation of some particles from the main part of the comet is produced by radiation pressure (or dynamically equivalent effects), and can be represented as a reduction in solar attraction, including of course complete reversal. Some properties of the resulting locus of particles (of different sizes and effective gravitation) leaving the comet at a single instant are discussed; some of the results obtained are applied to the particular case of Comet Arend–Roland. The theory provides a straightforward explanation of the apparent sunward ' spike ' observed for this comet in terms of a fan-shaped distribution of emitted particles observable only when viewed from positions in or very near its general plane.

---

1. *Introduction.*—The accretion theory of the origin of comets (1) suggests that the head of a comet consists of a diffuse swarm of particles which may not necessarily be of equal size. In the case of a long-period comet, mutual gravitation can prevent complete dispersal near aphelion, but near perihelion the Sun's differential gravitational field is so strong that the mutual attractions are negligible and the particles move virtually independently. The theory also requires that as the comet traverses the small section of its orbit on the perihelion side all the particles composing the head pass through the plane of general motion and thus tend to bunch more closely together. Because of this, collisions take place within the head, producing finer dust and possibly also gas which may, in part at least, go to form the tail. We shall consider the paths of these small dust particles after the collisions to obtain some indication of their distribution at any later time.

It is convenient to make some simplifying assumptions. We shall regard the motions as taking place in the plane of the comet's orbit, and initially assume that all the fine dust particles are produced simultaneously and instantaneously at a point of the comet's path which we shall take to be at an angular distance α from perihelion. Since the comet will be close to perihelion during the interval in which we are interested, each particle can be considered as moving independently, under the influence of the Sun's gravitational field and radiation pressure (or equivalent effects) only.

Both of these forces obey the inverse square law. However, radiation pressure varies as the surface area of the particle affected, while gravitation varies as its mass, that is (roughly) as its volume. Thus radiation pressure will have a greater comparative effect on small particles than large ones, and in the case of a comet some particles produced by internal collisions will be so tiny compared with those forming the head that they will be swept away from the main body, to form the tail.

If we treat the comet as a single moving point, as is usual when calculating an orbit, we can denote its corresponding central acceleration by μ/r^2. We shall take the orbit of this point to be parabolic. It will remain parabolic after the instant of collision if we assume that internal collisions have a negligible effect on the motion of the comet. At the instant of collision the dust particles will have the same speed and direction of motion as the head, and we shall assume these to remain unaltered by the collision* ; however, after this instant the central acceleration for each particle will be $\kappa\mu/r^2$, where κ is now essentially less than unity. The parameter κ will be different for different particles, but we shall assume that for any one particle it will remain constant in time. The motion of the comet itself will be given by the limiting case $\kappa = 1$; $\kappa = 0$ will correspond to a particle for which the gravitational force and the radiation pressure exactly cancel; $1 > \kappa > 0$ represents particles for which gravitational force is the stronger; and $0 > \kappa$ those for which radiation pressure is the stronger and the net effect an actual repulsion from the Sun. In any particular case the visible tail will correspond to values of κ between unity and some smaller (probably negative) value dependent on the density and composition of the tail material.

The problem may now be stated thus: if we consider a set of particles represented by values of κ in the continuous range $1 \geqslant \kappa > -\infty$, and given the specified conditions, what is the locus of these particles at some given time-interval after the comet passes the collision point?

2. *Orbits of particles for different values of* κ.—If the comet (considered as a particle) moves in a parabolic orbit with perihelion distance q, the equation of its path will be $2q/r = 1 + \cos\theta$ in polar coordinates, where r is the distance of the particle from the Sun and θ is measured from the initial line joining the Sun to perihelion.

The orbits of all the dust particles thus pass through the point $r = 2q/(1+\cos\alpha)$, $\theta = \alpha$. We can take the origin of the time scale at the instant the comet passes perihelion, and take the instant of collision to be $t = t'$. We have assumed that at $t = t'$ all dust particles have the same velocity as the comet, so the angular momentum per unit mass (in standard Keplerian orbit notation this is $h = r^2\dot{\theta}$), which is constant for an individual particle, must have the same value for every particle; that is, h is independent of κ. Using this fact and the starting conditions at $\theta = \alpha$ we obtain the orbit of the particle with central acceleration $\kappa\mu/r^2$ in the form

$$2q/r = \kappa + \cos\theta + (1-\kappa)\cos(\theta - \alpha). \tag{1}$$

To calculate the time t at which a particle starting at $\theta = \alpha$ at $t = t'$ reaches a point (r, θ) satisfying (1), we have, since $h = r^2\dot{\theta}$,

$$\int_{t'}^{t} h\,dt = \int_{\alpha}^{\theta} r^2\,d\theta. \tag{2}$$

We now introduce a new variable u, defined by

$$u^2 = 1 - 2q/r(1+\cos\theta) \tag{3}$$

noting that this quantity is dimensionless and identically zero for the case $\kappa = 1$. It is evident from (3) that $0 \leqslant u^2 \leqslant 1$; where necessary we shall take the positive root of u^2, so that $0 \leqslant u \leqslant 1$ also.

* It will be noted that many of the arguments used here do not necessarily apply only to comets possessing particle swarm structure. However, this particular approximation cannot easily be made in other theories.

Substituting for r in (2) from (3) and putting $s = \tan \frac{1}{2}\theta$, we obtain

$$h(t-t')/2q^2 = \int_a^s \frac{1+s^2}{(1-u^2)^2}\,ds \tag{4}$$

where $a = \tan \frac{1}{2}\alpha$.

In the case $\kappa = 1$, for which $u = 0$ always, this integrates immediately to

$$\sigma(t-t') = 2(s-a) + \tfrac{2}{3}(s^3 - a^3) \tag{5}$$

where $\sigma = h/q^2$ (independent of κ). However, since we have chosen the origin of the time scale to be the instant of perihelion passage, $t = 0$ when $s = 0$. Therefore, from (5),

$$\sigma t' = 2a + \tfrac{2}{3}a^3. \tag{6}$$

Consequently, in the case $\kappa = 1$,

$$\sigma t = 2s + \tfrac{2}{3}s^3. \tag{7}$$

3. *Locus at one instant of particles with different values of* κ.—Substituting for r in (3) from (1) for a particular value of κ, we obtain

$$\frac{u}{s-a} = \left(\frac{1-\kappa}{1+a^2}\right)^{1/2} \tag{8}$$

By substituting for t' from (6) and for s from (8) in (4) and integrating, we arrive at the result

$$\begin{aligned}\sigma t = {} & (1+a^2)(s-a)\left\{\frac{1}{1-u^2} + \frac{1}{2u}\log\frac{1+u}{1-u}\right\} \\ & + \frac{(s-a)^3}{u^2}\left\{\frac{1}{1-u} - \frac{1}{2u}\log\frac{1+u}{1-u}\right\} \\ & + \frac{2a(s-a)^2}{1-u^2} \\ & + 2a + \tfrac{2}{3}a^3.\end{aligned} \tag{9}$$

Since $0 < u < 1$ in (9) we can expand this expression in powers of u. It can be verified that if u is allowed to tend to zero the result (5) is obtained, corresponding to $\kappa = 1$.

4. *Predicted appearance of a tail.*—We could if we wished obtain explicitly an equation in polar coordinates of the locus of dust particles of varying κ by substituting in (9) the expressions that we have for u and s in terms of r and θ but independent of κ. However, it has been found more convenient to continue using our present variables.

To obtain a better indication of the distribution of dust particles after collisions within the head, it is necessary to consider what happens when such collisions take place at more than one instant. To do this it is only necessary to permit a to vary in (9).

Equation (9) is plotted in Fig. 1 in a representative case, using various values of t and assuming that the tail began to form at perihelion. For each value of t three values of a were chosen, corresponding to α equal to 0°, 15°, and 30°. A set of values of u were then taken and (9) solved for s in each case. A short programme was written for the IBM 650 computer for this purpose, equation (9) being solved for s by iteration.

5. *Some properties of a one-dimensional tail.*—The calculation of the angle between a tail produced at a single collision point and the path of the cometary head can readily be made. From (3), we have

$$x = r\cos\theta = q(1-s^2)/(1-u^2)$$

and

$$y = r\sin\theta = 2qs/(1-u^2)$$

whence, differentiating these with respect to s and dividing, we find

$$\frac{dy}{dx} = \frac{2s\lambda + 1 - u^2}{(1-s^2)\lambda - s(1-u^2)} \tag{10}$$

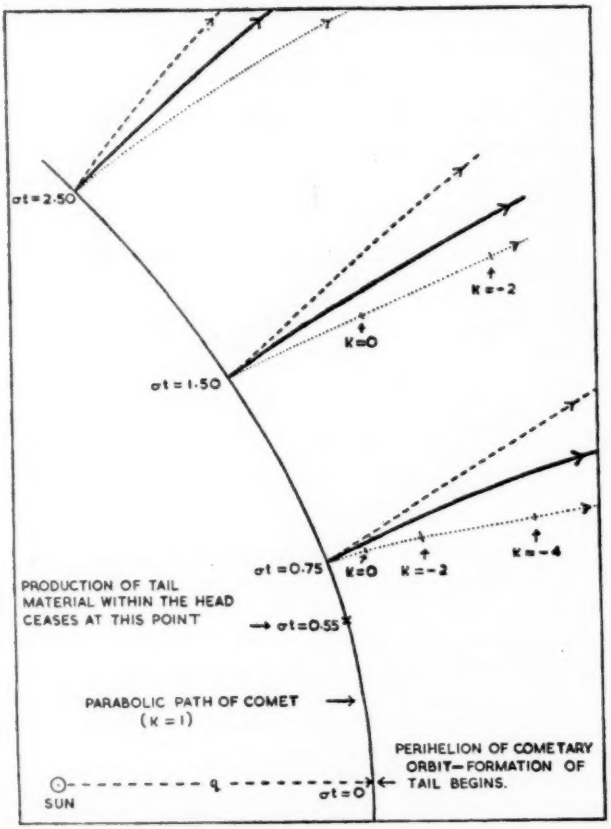


FIG. 1.—*The calculated distribution of dust particles at various times σt after perihelion passage. It is supposed that the tail begins to form at perihelion, and the dotted lines show later distributions of dust particles emitted from the head of the comet at this instant. The dashed lines show the distribution of the particles emitted at perihelion plus 30°, while the thick lines indicate the distribution of particles emitted at perihelion plus 15°. If collisions are considered to be taking place almost continuously between the extreme positions, most of the tail material can be expected to be concentrated about the 15° loci at any later time. Each of the lines represents a graph of equation (9) for fixed t and a and can be regarded as extending to infinity, although that part of each line further from the Sun than about 2·5q is not shown. The arrows show the direction in each locus of increasing* $u(\to 1)$ *and decreasing* $\kappa(\to -\infty)$.

where $\lambda = u\,du/ds$ (derived from (9)). Putting $u=0$, we have that the inclination of the tail at the head of the comet is

$$\tan\phi = \frac{2s\lambda_0 + 1}{(1-s^2)\lambda_0 - s} \tag{11}$$

where λ_0 is the value of λ at $u=0$. The inclination of the path at the point (r, θ) is $-1/s$, so the tangent of the angle required is $\lambda_0/(s\lambda_0+1)$. To find λ_0 we differentiate (9) with respect to s and put $u=0$. This reduces to

$$\lambda_0 = -\frac{15(1+s^2)}{2(s-a)\{10+a^2+3as+6s^2\}}$$

and the expression for the tangent of the angle between the tail and the direction of motion of the head is obtained in the form

$$\frac{15(1+s^2)}{3s^3+6as^2+(4a^2-5)s+20a+2a^3}. \tag{12}$$

In any particular case this will give the angle at which the tail leaves the head, measured from the direction of motion of the head, at any time after the comet passes a single collision point for values of $s > a$. The required value of s corresponding to a particular time t (measured, of course, from perihelion passage) is found by solving (7) for s. As an example we can take the simple case $a=0$, that with the collision point at perihelion. Since the angle between the tail and the direction of motion of the head is a right angle when the cubic denominator vanishes, we immediately see that in this case this is so for $s=(5/3)^{1/2}$, which corresponds to an angle θ of roughly $104\frac{1}{2}°$. Equation (7) gives the value of σt as $(28/9)(5/3)^{1/2}$ or about 4·0; up to this time the cubic denominator is negative, so the angle is obtuse. For all σt greater than this value the angle is acute, and tends to zero as $s\to\infty$, that is as the comet moves away from the Sun towards aphelion. This is of course an expected result and accords with observation.

To find the behaviour of the locus as $r\to\infty$ we merely have to investigate equation (9) for $u\to 1$. For simplicity we again take the special case $a=0$, but similar arguments are applicable in the general case. Writing

$$A = \frac{1}{1-u^2} + \frac{1}{2u}\log\frac{1+u}{1-u}$$

and

$$B = \frac{1}{u^2}\left\{\frac{1}{1-u^2} - \frac{1}{2u}\log\frac{1+u}{1-u}\right\}$$

equation (9) reduces to

$$\sigma t = sA + s^3B. \tag{13}$$

It can be easily verified that $A > B > 0$ for all u in the permissible range, and that as u tends to unity both A and B diverge. However, since s, A, and B are all positive and at any instant σt is constant for all u, sA must be less than σt for all u. Thus s must tend to zero in order to preserve this inequality. Consequently, since $A > B > 0$, s^3B tends to zero. By a similar argument it may be shown that the logarithmic part of sA tends to zero also, and so

$$s/(1-u^2)\to\sigma t \quad \text{as} \quad u\to 1.$$

Now (3) may be rewritten in the form

$$r/(1+s^2) = q/(1-u^2) \tag{3a}$$

from which follows

$$y \equiv r\sin\theta = 2qs/(1-u^2).$$

This measures the distance y of any point on the locus from the initial line. Thus as $r \to \infty$

$$y \to 2q\sigma t = 2Vt$$

where V is the velocity of the comet itself at perihelion. However, since the particle for which $\kappa = 0$ (i.e. that for which the gravitational force and the radiation pressure exactly cancel) travels in a straight line perpendicular to the initial line with constant velocity V, we see that the end of such a tail moves away from the axis with twice the velocity of the free particle.

From (10) it is easily demonstrated that the inclination of the tail is positive for $\kappa < 0$. The conclusion is that the tail at infinity is parallel to the initial line.

Similarly, in the general case the end of the tail moves away from the line joining the Sun and the collision point twice as fast as does the free particle. In fact, as $r \to \infty$

$$y' \equiv r \sin(\theta - \alpha) \to 2V'(t - t')(1 + a^2)^{-1/2}$$

where V' is the velocity of the comet at the collision point. $V'(1 + a^2)^{-1/2}$ is of course the component of V' perpendicular to the radius vector.

6. *Applications to Comet Arend–Roland.*—Some of the preceding results have particular application to Comet Arend–Roland 1956 h. It has been pointed out (**2**, **3**) that at the time of the appearance of the anomalous sunward tail the Earth was passing through the plane of the cometary orbit. According to the elements compiled by Candy (**4**), the Earth was at the ascending node of the orbit on 1957 April 25, at an angular distance of about 51° 15′ past the cometary perihelion. Taking the Earth's distance from the Sun at this time to be one astronomical unit, a rough calculation shows that the comet itself was then at an angular distance of about 82° past perihelion.

By differentiating (11) with respect to a, and making use of the expression we have for λ_0, it is easy to show that at any particular point of the orbit defined by keeping s constant the inclination $\tan \phi$ of a one-dimensional tail at the head of a comet is a monotonically increasing function of a. Now the gradient of the line joining the Earth and the comet on 1957 April 25 was about 0·42. Putting $s = \tan 41°$ and $\tan \phi = 0{\cdot}42$ in (11) gives a value of a of about 0·27, which corresponds to a negative value of α of about $-30°$. Since $\tan \phi$ is a monotonically increasing function of a, this means that if the tail began to form at an earlier stage than 30° before perihelion passage, the particles emitted from the head at the earliest collisions points will lead to values of $\tan \phi$ smaller than 0·42. The situation is illustrated in Fig. 2, which demonstrates clearly that, at the time that the Earth passed through the plane of the orbit, that part of the tail which had been produced by collisions occurring before the comet reached an angular distance of 30° before perihelion would have appeared from the Earth to be on *the sunward side of the comet's head.* That part produced by later collisions would appear as a normal tail. The similarity between this diagram and Armstrong's conjectural figure (**3**) is clear.

The work of Lyttleton has shown that it is not unreasonable to expect some collisions to take place so early (**1**, p. 117), although most collisions can be expected close to perihelion, which explains the simultaneous appearance of a normal tail. It can also be seen that, since the Earth was at the ascending node of the cometary orbit on 1957 April 25, the 'spike' would have appeared to point

slightly to the south of the Sun just before the node was reached, and slightly to the north just afterwards. This is again in accordance with observation (**2**).

At first sight it might be expected that, as in the case of the rings of Saturn, the anomalous tail would be too narrow to be visible to an observer in its own plane.

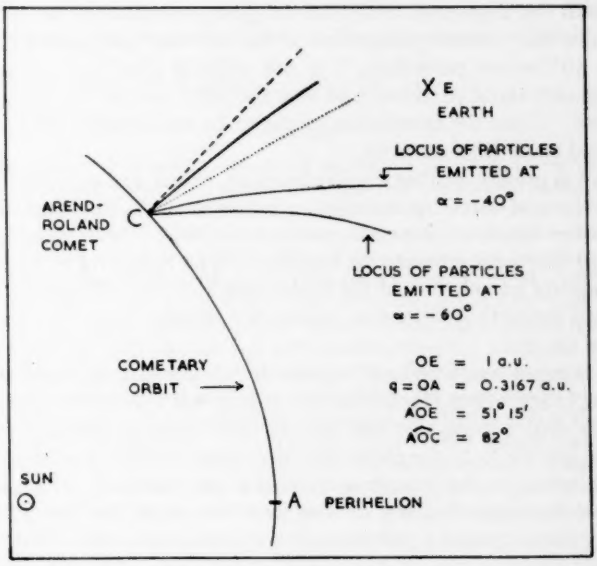
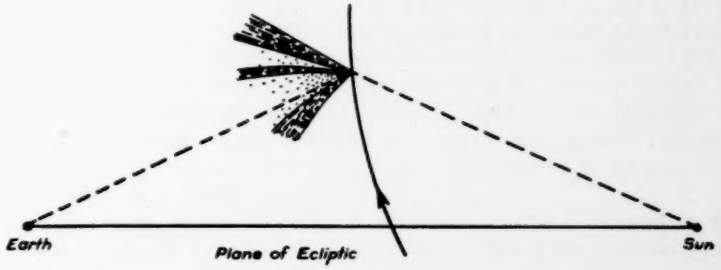


FIG. 2.—*The situation as the Earth passed through the plane of the orbit of Comet Arend–Roland on* 1957 *April* 25. *The tail as given by the IBM* 650 *programme (see Section* 4) *is shown as in Fig* 1, *together with two loci of particles emitted before the comet reached* 30° *before perihelion. The figure demonstrates how such particles appear on the sunward side of the head, while particles emitted later go to form the normal tail. The figure by Armstrong referred to in the text (Section* 6) *is reproduced below for comparison.*



However, we have throughout taken the problem to be two-dimensional to bring it within the range of analysis. Considerable dispersion of the tail outside the orbital plane (but with the greatest density at the plane) is to be expected, since the cometary head is not the point-particle we have assumed it to be, but a swarm of particles, with many of these necessarily having small motions perpendicular to the plane of general motion. Also, we have not taken into consideration any

additional components of velocity of the ejected particles perpendicular to the orbital plane, whereas with random collisions we must expect such components to add to the dispersion of the tail.

Nevertheless, a cometary head can subtend only a small angle at the sun, so the consequent dispersion perpendicular to the plane is not likely to be large compared with the dispersion in the orbital plane revealed by our analysis. In addition, since only a small proportion of the tail can be expected to be formed earlier than 30° before perihelion, it is not unlikely that, to an observer in the orbital plane, only those particles in or near the plane will be dense enough to give a visible effect. Thus the narrowness of the spike in contrast to the dispersion of the tail should cause little surprise.

The author is greatly indebted to Professor H. Bondi and to Dr R. A. Lyttleton for much advice and encouragement in the preparation of this paper, and to Mr. J. K. Iliffe and the Applied Science Department of IBM United Kingdom Limited for generously providing computing facilities. The figure from Dr Armstrong's paper (**3**) appears by courtesy of Dr Armstrong and the Editors of *Observatory*, to whom also the author wishes to express his thanks.

Since this paper was prepared, a paper by Osterbrock (**5**) on the tails of the comets Baade (1954 h) and Haro-Chavira (1954 k) has appeared. Calculations are shown which demonstrate that the tails of these comets, although lying in the orbital plane, did not lie in the radial direction from the Sun, and explanations are suggested in terms of the composition of the tail material. It seems that the present paper has some bearing on this problem, since the preceding analysis shows clearly that a cometary tail does not necessarily lie in the radial direction.

King's College, London:
1958 *October* 22.

References

(1) R. A. Lyttleton, *The Comets and their Origin* (Cambridge University Press, 1953).
(2) F. L. Whipple, *Nature*, **179**, 1240, 1957.
(3) E. B. Armstrong, *Observatory*, **77**, 135, 1957.
(4) M. P. Candy, *I.A.U. Circ. No.* 1585 (1957 Feb. 20).
(5) D. E. Osterbrock, *Ap. J.*, **128**, 95, 1958.

COLLISION BROADENING AND SHIFT IN THE RESONANCE LINE OF CALCIUM

W. R. Hindmarsh

(Communicated by the Director, University Observatory, Oxford)

(Received 1958 November 21)

Summary

The collision shift and broadening of the line Ca 4227 A due to an external pressure of helium have been measured. The line was formed in absorption by passing white light through calcium vapour in the presence of helium at various pressures of less than 1 atmosphere. The spectral resolution was provided by a Babcock grating used in the new Oxford solar spectrograph. The half-intensity damping width of the line was found to be $1{\cdot}72 \pm 0{\cdot}12 \times 10^{-20}$ cm^{-1} per atom per cc of helium, and the shift $0{\cdot}05 \pm 0{\cdot}04 \times 10^{-20}$ cm^{-1} per atom per cc towards the violet. The ratio of broadening to shift is therefore $\geqslant 19$, compared with the value of 2·76 predicted by the Lindholm theory. The shift is also predicted to be towards the red. Possible explanations of the discrepancy are discussed and it is shown that the short-range repulsive forces between calcium and helium atoms must be involved as well as the long-range van der Waals forces. With the use of Lindholm's theory, coefficients for both types of force are evaluated from the measurements. Finally it is shown that the solar damping coefficient for Ca 4227 A is $< 2{\cdot}2 \times 10^{-3}$ cm^{-1} at $\tau = 0{\cdot}1$, and that the collision shift makes a negligible contribution to the solar red shift.

Introduction.—Collision broadening is an important process in the formation of stellar absorption lines, and a knowledge of the damping coefficients of such lines would remove an important unknown from the theory of line contours. Collision shifts have been discussed as a possible explanation of the fraction of the red shift of solar lines which is unaccounted for by the Einstein gravitational shift (**1**, **13**). The work described in this and the following paper forms the first two of a series of investigations into collision effects in atomic spectra. Stellar absorption lines are formed, in general, in atmospheres of rather low pressure, and the present investigations are therefore aimed at the pressure region below 1 atmosphere. In stellar atmospheres, collision broadening is probably due in many cases to collisions of the radiating atoms with both neutral and charged particles. The experiments here described were designed to measure neutral particle broadening and shift only.

The most useful theory of collision effects in spectra developed to date is that due to Lindholm (**9**, **10**). A similar theory was given independently by Foley (**4**). This theory gives the following expression for the absorption coefficient $\kappa(\nu)$ of an atom absorbing in the presence of N perturbing atoms per cc:—

$$\kappa(\nu) = \frac{\text{constant}}{(\nu - \nu_0 - \beta)^2 + \gamma^2}$$

β and γ are constants, and ν is in cm^{-1}. This expression represents a symmetrical contour of damping shape and half intensity width 2γ shifted with respect to the

line centre at $\nu = \nu_0$ by an amount β. Assuming a van der Waals interaction—an attractive force whose potential is proportional to the inverse sixth power of distance between the radiating and perturbing atoms—the values of β and γ are

$$\beta = -4{\cdot}90 \times 10^{-7} C_6^{2/5} v^{3/5} N$$

$$\gamma = 6{\cdot}75 \times 10^{-7} C_6^{2/5} v^{3/5} N;$$

v is the mean relative velocity of the colliding and radiating atoms; C_6 is the interaction constant defined by $\delta\nu = -C_6/r^6$, $\delta\nu$ being the change in wave number of the radiation caused by a perturbing atom at distance r. Lindholm (**9**, **10**) and also Unsöld (**15**) have discussed the range of validity of this expression. They find that the approximations involved are good for those parts of the line contour for which

$$(\nu - \nu_0) \ll 6{\cdot}09 \times 10^{-14} \frac{v^{6/5}}{C_6^{1/5}}.$$

Substituting for C_6, this inequality becomes

$$(\nu - \nu_0) \ll 5{\cdot}00 \times 10^{-17} v^{3/2} \left(\frac{N}{\gamma}\right)^{1/2}. \qquad (1)$$

γ/N frequently lies in the range $10^{-19} - 10^{-20}$ cm^{-1} per atom per cc, and $v \sim 2 \times 10^5$ cm sec^{-1} for most laboratory experiments with light gases. In these circumstances the condition is $(\nu - \nu_0) \ll 45$ cm^{-1}, or about 8 A at 4227 A. The low pressure formula for $\kappa(\nu)$ may therefore only be applied to the comparatively narrow lines formed at particle densities of less than about 2×10^{19} per cc, for which $\gamma \ll 45$ cm^{-1}.

Little previous experimental work on collision broadening and shift has been carried out at these low pressures. Kleman and Lindholm (**6**) have measured the broadening and shift of the sodium D lines produced by argon at low pressures, and found that the ratio $2\gamma/\beta$ was about $-4{\cdot}0$ as compared with the theoretical value of $-2{\cdot}76$. Kusch (**7**) has recently measured some iron lines broadened by atomic hydrogen in conditions which just satisfy the criterion, and finds that the ratio $2\gamma/\beta$ varies from $-2{\cdot}5$ to $-3{\cdot}6$ for different lines. Kusch, however, claims an accuracy of only ± 30 per cent for the ratio $2\gamma/\beta$ and ± 40 per cent for his values of γ/N. He has also measured the broadening only of the resonance line of calcium, $\lambda 4227$, in an atmosphere of molecular hydrogen. Lang (**8**) has measured the broadening only of some neon and argon lines excited in constricted glow discharges at pressures below 2 cm mercury.

It seems, therefore, that accurate measurements of collision broadening and shift would be of considerable value, both for the adequate testing of the Lindholm-Foley theory and for the elucidation of certain astrophysical problems. There are at least two possible methods of making such measurements. First, the line may be measured in absorption (with the use of a simple absorption tube for resonance lines of volatile elements, or a King furnace for other lines). This has the advantages that the damping coefficient of an absorption line is comparatively easy to determine and that the line is produced in well defined conditions, and in the virtual absence of charged particles. The disadvantages are that, even with the best available diffraction gratings, pressures of a substantial fraction of 1 atmosphere are necessary to obtain accurate measurements, and that lines of excitation potential greater than about 10 volts cannot be obtained in a King furnace. The second possible method is to excite the line in emission in a gas discharge and use interferometric techniques for the

measurements. With this method, very low pressures may be used, though in that event the broadening is exceedingly difficult to measure. Further, the presence of charged particles in the discharge makes necessary some investigation of their influence on the line. This paper provides an example of the former type of experiment, and the following paper an example of the latter.

Experimental method.—The resonance line of calcium, λ 4227 A, formed in absorption in an atmosphere of helium, was examined, and the collision shift and broadening determined. The calcium and helium were contained in a steel absorption tube 15 cm long and 2 cm in diameter, which was heated to a temperature of about 400 °C by means of an electric furnace. The temperature for a given heating current was measured in a separate experiment when a (platinum)–(platinum-rhodium) thermocouple was inserted in the tube. This could not be done in the presence of calcium vapour, as the latter reacted chemically with the thermocouple wires. At a temperature of 400 °C the vapour pressure of the calcium was sufficient to give an absorption line of fairly small but easily measurable central intensity. The ends of the tube were cooled by means of water jackets, and apiezon wax was used to attach glass windows. The influence of the non-uniform temperature in the absorption tube on the line contours was estimated. Errors in the measured damping coefficient due to this cause are not more than 3 per cent. Physically this small effect arises because of the very rapid variation of vapour pressure with temperature, so that only a thin layer in the hottest part of the tube is effective in forming the absorption line. The tube was connected to a mercury manometer and to a cylinder containing helium through a liquid-oxygen cooled charcoal trap. The whole system could be evacuated by means of a rotary-oil pump. The purpose of the charcoal trap was to ensure that no trace of oxygen entered the absorption tube, as calcium reacts very rapidly with it to form calcium oxide, a stable and refractory substance.

A tungsten strip-filament lamp provided a source of continuous radiation. The light passed through the absorption tube in a collimated beam, then through a combination of colour filters to isolate the spectral region required, and an image of the strip was formed in a plane in which a rhodium step filter could be placed for calibration purposes. Finally, the filament was re-imaged on the slit of a plane-grating Littrow spectrograph of 12 m focal length, the new Oxford solar spectrograph. The grating was ruled by H. W. Babcock. It has 600 lines per mm and an aperture of 20 cm. It was used in the sixth order, where the blaze was effective, and the dispersion of about 7 mm per A was measured from iron-arc spectra taken for the purpose. The spectrograph slit was covered with a diaphragm which divided the slit length into four sections, each of which could be opened or closed by means of a shutter. On the lower two of these divisions fell the image of the strip filament lamp, while on the upper pair light from a G.E.C. cadmium lamp could be directed by means of a right angled prism. The cadmium green line λ 5086 A in the fifth order fell about 5 cm from the calcium line λ 4227 A in the sixth order, and the long-wavelength hyperfine-structure component of this cadmium line was used as a reference line to evaluate spurious shifts due to changing atmospheric pressure and other causes.

The experimental procedure was as follows. Helium was admitted to the tube at one of the three pressures 6 cm, 40 cm, or 75 cm of mercury, the tube already containing calcium. The tube was maintained at a temperature of about 400 °C. The continuous spectrum of a strip-filament lamp was then exposed

simultaneously with the cadmium green line, each through one of the four divisions of the slit length. Then the shutters were moved to open the other two divisions, the helium pressure changed to one of the other two values already mentioned, and a similar exposure taken. One of the two helium pressures was always 6 cm of mercury. Exposures for plate calibration were made in the usual way, with the use of a calibrated rhodium step filter (**3**). With Ilford Zenith plates a time of 2 hours was required for each exposure when the slit width was 0·12 mm. 5 plates were obtained for the measurement of the shift between 6 cm and 40 cm mercury pressure, and 5 for the shift between 6 cm and 75 cm. 6, 5 and 4 spectra were used to evaluate the broadening at the pressures 6 cm, 40 cm and 75 cm respectively.

Measurement of the shift. To evaluate the wavelength shift of the calcium line caused by changing the pressure of helium in the absorption tube, the positions of the calcium and cadmium lines were measured on a modified Hilger measuring micrometer whose plateholder could be moved in two mutually perpendicular directions. The appearance of a plate is shown diagrammatically in Fig. 1, where the shifts have been exaggerated for the sake of clarity. The

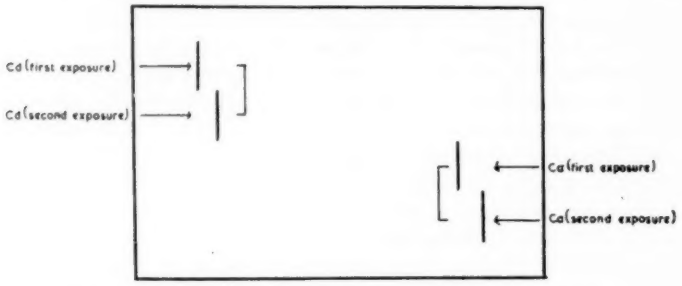


FIG. 1.—*Diagrammatic illustration of the appearance of a plate.*

shift in mm between the two cadmium exposures was subtracted from the shift in mm between the two calcium exposures to give the true collision shift of the calcium line, which was converted into cm^{-1} and A with the use of the measured value of the dispersion. Mechanical displacements of the plate, slit or grating during an exposure are clearly eliminated by this procedure. Consider now the effect of changing atmospheric density. If the latter changes by $\Delta\rho$ then the changes in wavelength of the calcium line and of the cadmium line are given by

$$\frac{\Delta\lambda_{Ca}}{\lambda_{Ca}} = (\mu_{Ca} - 1)\frac{\Delta\rho}{\rho} \quad \text{and} \quad \frac{\Delta\lambda_{Cd}}{\lambda_{Cd}} = (\mu_{Cd} - 1)\frac{\Delta\rho}{\rho},$$

where μ_{Ca} and μ_{Cd} represent the refractive indices of air at λ_{Ca} and λ_{Cd} respectively. Then

$$\Delta\lambda_{Ca} = \Delta\lambda_{Cd}\frac{\lambda_{Ca}}{\lambda_{Cd}}\cdot\frac{\mu_{Ca} - 1}{\mu_{Cd} - 1}$$

and if the dispersions of the spectrograph at the two wavelengths are D_{Ca} and D_{Cd} mm/A and the shifts in mm on the plate are Δx_{Ca} and Δx_{Cd}

$$\Delta x_{Ca} = \Delta x_{Cd}\frac{\lambda_{Ca}}{\lambda_{Cd}}\frac{(\mu_{Ca} - 1)}{(\mu_{Cd} - 1)}\frac{D_{Ca}}{D_{Cd}}$$
$$= \Delta x_{Cd} \text{ to an accuracy of 1 per cent.}$$

Hence the spurious shifts due to changes in air density are eliminated by subtracting the cadmium shift in mm from that of the calcium shift in mm. Further errors may arise because of lack of perpendicularity of the two stage motions and inaccuracy in the alignment of the plate on the microscope. These were eliminated by ensuring that the distance through which the microscope was moved parallel to the length of the spectral lines (marked by the square brackets in Fig. 1) was the same for both the cadmium and the calcium shifts. An estimate was made of the influence of curvature of the spectral lines, and this was found to be negligible. Each plate was measured twice, and inverted between the two measures. There was no significant difference between these two measures.

Table I shows the results obtained. The quoted error is in each case the standard deviation of the mean. The pressure has been converted into particle density with the use of the measured temperature, which was 400 °C for the 6 cm pressure exposures, 392 °C for the 40 cm and 416 °C for the 75 cm.

TABLE I

Shift between $0{\cdot}86\times10^{18}$/cc and $5{\cdot}80\times10^{18}$/cc atoms of helium		Shift between $0{\cdot}86\times10^{18}$/cc and $10{\cdot}48\times10^{18}$/cc atoms of helium	
10^{-3} cm^{-1}	mA	10^{-3} cm^{-1}	mA
+15·1	−2·7	+10·5	−1·9
+ 5·6	−1·0	+ 0·5	−0·1
−24·0	+4·3	+ 7·3	−1·3
+ 7·3	−1·3	+15·1	−2·0
− 2·2	+0·4	+ 3·9	−0·7
Mean + 0·4 ± 3·4	−0·1 ± 0·6	Mean + 7·4 ± 2·3	−1·2 ± 0·4

From these results we find a mean value of the quantity β/N, assuming that the shift between x atoms of helium per cc and y atoms of helium per cc is equal to that between $x-y$ atoms per cc and zero atoms per cc, provided $y \ll x$. The result is

$$\beta/N = 0{\cdot}05 \pm 0{\cdot}04 \times 10^{-20}\ \text{cm}^{-1}\ \text{cm}^3.$$

Measurement of the damping coefficient.—The calcium absorption lines produced at the three pressures of helium were measured with the use of a Moll-type microphotometer (**12**). The characteristic curve of each plate was plotted from microphotometer tracings of the step-wedge exposures. The curves for all plates but one were averaged together. The curve of the remaining plate was used for that plate, since it was significantly different from the others. Two tracings of each calcium exposure were taken from portions of the absorption line, and photographic density reduced to intensity of light in the usual way.

It was assumed that the line contours could be expressed in the form $I(\nu) = I_0 e^{-\alpha(\nu)}$ where $\alpha(\nu)$ is the atomic absorption coefficient of calcium modified by the apparatus function of the spectrograph and multiplied by the number of atoms per cm^2 of the absorption tube cross section. $\alpha(\nu)$ is the optical depth of the calcium vapour at frequency ν, and it was computed from the observed $I(\nu)$. The resulting curve was fitted to a Voigt profile at the 0·5 and 0·1 intensity widths with the use of the tables given by van de Hulst and Reesinck (**5**), and the half intensity widths of the damping and Gaussian components determined. The fit to a Voigt profile was in all cases better than 3 per cent of the maximum value of $\alpha(\nu)$. A typical curve for each of the pressures is given in Fig. 2, and

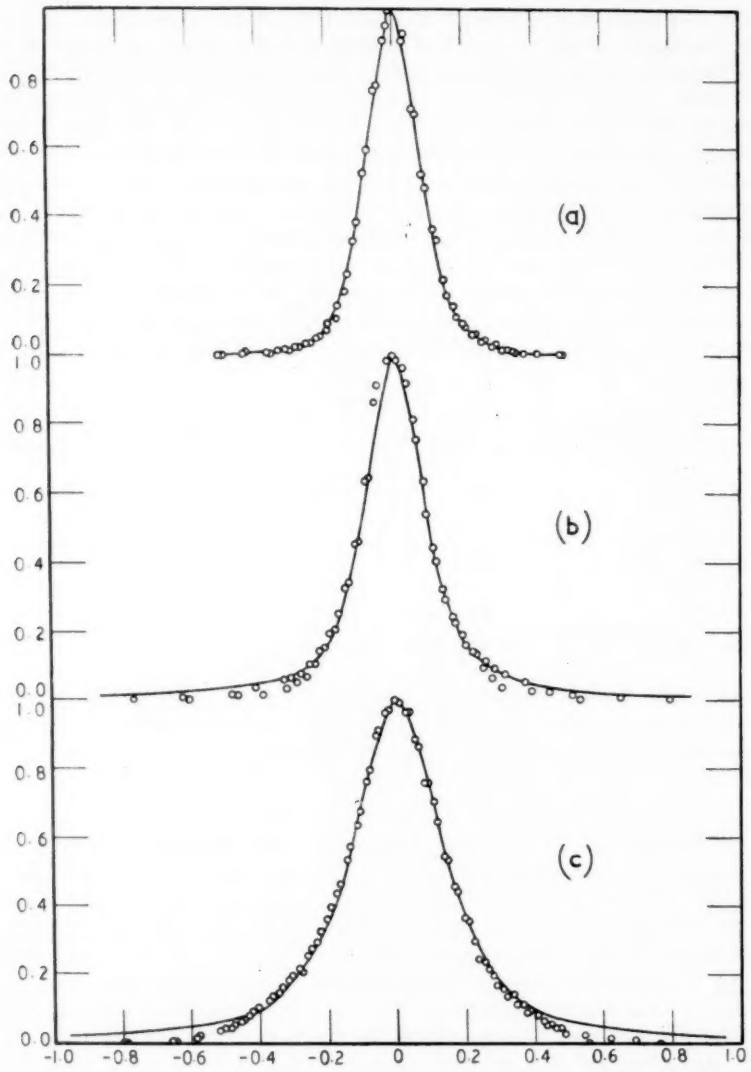


FIG. 2.—*Observed values of* $\alpha(\nu)$, *together with the fitted Voigt profiles, for a typical plate for each pressure.* (*a*) 6 *cm helium pressure;* (*b*) 40 *cm helium pressure;* (*c*) 75 *cm helium pressure. Ordinates are on an arbitrary scale, and abscissae are in* cm^{-1} *measured from the line centre.*

points from both the tracings for the plate chosen—that at the head of its respective column in Table II—plotted. Table II shows the individual plate measures, and the errors are standard deviations of the mean. There are no significant differences among the Gaussian half-intensity widths measured at the three different pressures. A plot of the damping half-intensity width against particle

TABLE II

6 cm helium pressure $0{\cdot}86 \times 10^{18}$ atoms helium/cc		40 cm helium pressure $5{\cdot}80 \times 10^{18}$ atoms helium/cc		75 cm helium pressure $10{\cdot}48 \times 10^{18}$ atoms helium/cc	
Damping width cm^{-1}	Gaussian width cm^{-1}	Damping width cm^{-1}	Gaussian width cm^{-1}	Damping width cm^{-1}	Gaussian width cm^{-1}
0·060	0·148	0·152	0·147	0·216	0·202
0·089	0·142	0·175	0·195	0·225	0·173
0·076	0·102	0·156	0·093	0·241	0·175
0·083	0·129	0·138	0·099	0·271	0·065
0·048	0·128	0·138	0·114		
0·086	0·094				
$0{\cdot}073 \pm 0{\cdot}006$	$0{\cdot}124 \pm 0{\cdot}008$	$0{\cdot}152 \pm 0{\cdot}006$	$0{\cdot}130 \pm 0{\cdot}017$	$0{\cdot}238 \pm 0{\cdot}010$	$0{\cdot}154 \pm 0{\cdot}026$

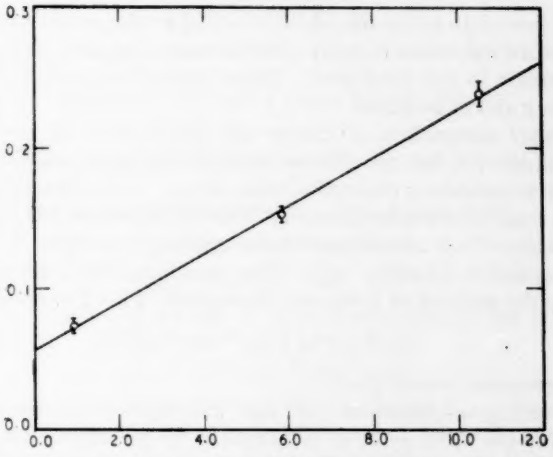


FIG. 3.—*Plot of damping half-intensity width against helium particle density. The straight line is the least squares line through the three observed points. Ordinates are in* cm^{-1} *and abscissae in units of* 10^{18} *atoms helium per cc.*

density of helium is shown in Fig. 3. The equation of the least squares line through the three points is

$$\Delta\nu = 1{\cdot}72 \times 10^{-20} N + 0{\cdot}056.$$

There is every reason to believe that the collision damping coefficient is zero at zero particle-density of helium, for " self " broadening of the calcium must be negligible at the very small vapour pressures of calcium involved in this experiment. The residual damping width of $0{\cdot}056\,cm^{-1}$ at $N=0$ is therefore attributed to the apparatus function of the spectrograph, and may be subtracted from the observed damping width to give the true collision width, so that the result is

$$2\gamma/N = 1{\cdot}72 \pm 0{\cdot}12 \times 10^{-20}\,cm^{-1}\,cm^{3}.$$

The mean value of all measures of the Gaussian half-intensity width was

$$\Delta\nu_G = 0{\cdot}134 \pm 0{\cdot}010\,cm^{-1}.$$

This is due to the Doppler width of the absorption line and the apparatus function

of the spectrograph. As a check on the plausibility of this figure and the figure used above as the damping component of the apparatus function, the data given by Pierce (**11**) for the apparatus function of a Babcock grating were assumed for the grating used in the present experiment, and the Gaussian and damping components to be expected here were computed. The expected damping half width was $0{\cdot}046\,\mathrm{cm}^{-1}$, compared with that observed of $0{\cdot}056\,\mathrm{cm}^{-1}$, while the Gaussian apparatus half width added to the calculated Doppler width was found to be $0{\cdot}166\,\mathrm{cm}^{-1}$, compared with the observed value of $0{\cdot}134\,\mathrm{cm}^{-1}$. The agreement between the expected and observed values is as close as can be hoped for with the assumptions involved.

Discussion of the results.—The simplest test of the Lindholm theory is to compare the experimental and theoretical values of $2\gamma/\beta$, the ratio of broadening to shift. The value obtained from the above results is

$$19 < 2\gamma/\beta < \infty$$

while the theoretical value is 2·76. Furthermore, the observed shift is in the opposite direction to that predicted. There is therefore a marked disagreement between theory and experiment.

As a further comparison of theory and experiment, an attempt has been made to evaluate the van der Waals attraction between calcium and helium atoms, and so to estimate a theoretical value of 2γ. The expression for C_6 due to London was used in the form given by Unsöld (equations 82, 49 of **15**). The results of Hartree-Fock calculations of the oscillator-strengths were used where these were available (Trefftz, **14**). The remaining oscillator-strengths were calculated by the method of Bates and Damgaard (**2**). The result is

$$2\gamma/N = 1{\cdot}74 \times 10^{-20}\,\mathrm{cm}^{-1}\,\mathrm{cm}^{3}$$

so that the agreement here is good.

Before seeking explanations of the discrepancy between theory and experiment for the ratio $2\gamma/\beta$, it is necessary to investigate whether the low pressure approximation is valid in the conditions of the experiment. Substitution in equation (1) gives the condition for validity as

$$\nu - \nu_0 \ll 48\,\mathrm{cm}^{-1}.$$

Since the values of $\alpha(\nu)$ in these experiments were in all cases indetectably small for $(\nu - \nu_0) > 1\,\mathrm{cm}^{-1}$, the criterion is well satisfied.

There are two possible explanations for the major discrepancy revealed by these measurements. First, the Lindholm theory may be incorrect even at low pressures. The theory treats the collision process classically. Lindholm has, however, considered the modifications introduced by a quantum mechanical treatment of the collisions in the special case of the sodium D lines broadened by argon, and has found that only quite small corrections to the classical expressions are needed. It seems unlikely that the case of the calcium resonance lines broadened by helium will be very different, although this explanation of the discrepancy cannot be wholly discounted. The second possible cause of the disagreement is that the interaction of the calcium and helium atoms may not be described by a van der Waals (r^{-6}) law. To investigate this, it is convenient to evaluate from the experimental value of $2\gamma/N$ the collision cross-section σ, for this gives the order of magnitude of the distance of closest approach of the

calcium and helium atoms. Now $\gamma/N = v\sigma/c$, from which we find

$$\sigma = 1{\cdot}3 \times 10^{-15}\ \text{cm}^2,$$

and the collision radius is $2{\cdot}1 \times 10^{-8}$ cm. Since the radius of the helium atom is about 1·0 A and that of the calcium atom about 2·3 A, the collisions effective in broadening the line are rather close collisions, and at these small interatomic distances the repulsive forces between the atoms must be operative. Thus the assumption of an r^{-6} interaction is probably invalid. The repulsive forces contribute to the broadening, and produce a shift in the opposite direction to the attractive r^{-6} forces. It is clear, therefore, that the observations can be accounted for qualitatively in this way. More quantitatively, we may carry through the Lindholm theory for an interaction of the form

$$\delta\nu = -\frac{C_6}{r^6} + \frac{C_{12}}{r^{12}}.$$

An r^{-12} form for the repulsive forces is frequently adopted in molecular theory. When the Lindholm theory is developed for this interaction, we find

$$\frac{\gamma}{N} = 6{\cdot}75 \times 10^{-7}\, C_6^{2/5} v^{3/5} + 1{\cdot}94 \times 10^{-9}\, C_{12}^{2/11} v^{9/11}$$

$$\frac{\beta}{N} = -4{\cdot}90 \times 10^{-7}\, C_6^{2/5} v^{3/5} + 0{\cdot}57 \times 10^{-9}\, C_{12}^{2/11} v^{9/11}.$$

Substitution of the experimental values for γ and β then yields

$$C_6 = 5{\cdot}31 \times 10^{-45}\ \text{cm}^{-1}\,\text{cm}^6$$

$$C_{12} = 1{\cdot}20 \times 10^{-87}\ \text{cm}^{-1}\,\text{cm}^{12}$$

compared with the calculated value of C_6 (corresponding to the calculated $2\gamma/N$ given above)

$$C_6 = 2{\cdot}13 \times 10^{-43}\ \text{cm}^{-1}\,\text{cm}^6.$$

There appears to be no simple method of calculating C_{12}. There is not now good agreement between the calculated and measured values of C_6. The position is that we may retain the Lindholm theory by modifying the force law between calcium and helium atoms and accepting the disagreement with the calculated value of C_6, or we may retain a pure r^{-6} law (though the measured collision cross-section makes this procedure of very dubious validity) with the good agreement between calculated and observed values of C_6, and suppose that the Lindholm theory is correct for the broadening but incorrect for the shift of the line. In either case the present situation is unsatisfactory, and can probably only be improved by making an extensive series of measurements on a variety of spectral lines.

It remains now to discuss the astrophysical significance of the results. In stellar atmospheres, the collision effects in Ca 4227 A are due almost entirely to hydrogen atoms and electrons, while the measurements described in this paper refer to helium atoms. If we assume that the force between the atoms is of the van der Waals type (r^{-6}) for both helium and hydrogen, then the ratio C_6 (hydrogen)/C_6 (helium) has been calculated by Unsöld (**15**) to be approximately 3·0. Taking a layer in the solar atmosphere where the density of hydrogen is 5×10^{16} atoms per cc (optical depth $\tau = 0{\cdot}1$) and the temperature 5000 °K, then using the measured value of γ (helium), we find

$$\gamma(\text{hydrogen}) = 1{\cdot}9 \times 10^{-3}\ \text{cm}^{-1}.$$

This computation takes no account of the probable importance, revealed by the measurements, of the repulsive forces between calcium and helium atoms. There is no simple method of comparing these forces for hydrogen and helium. However, since the range of the repulsive forces in the hydrogen case must be smaller (because of the smaller radius of the hydrogen atom) while the strength of the attractive van der Waals forces must be greater (because of the larger polarizability of hydrogen) the repulsive forces are expected to be relatively less important in the hydrogen case. Thus the value for γ(hydrogen) is an upper limit. The contribution of the electrons may be estimated from the known Stark coefficient of Ca 4227 A, taking the electron density in the layer concerned to be 6×10^{12} cm^{-3} and using the Lindholm theory. The result is $\gamma(\text{electron}) = 2{\cdot}8 \times 10^{-4}$ cm^{-1}. An upper limit for the total damping coefficient in this particular layer of the solar atmosphere is then given by

$$\gamma(\text{Sun}) = 2{\cdot}2 \times 10^{-3}\,\text{cm}^{-1}.$$

An upper limit for the solar collision shift may be found by using the measured γ(helium), calculating β(helium) from the Lindholm theory, converting to the conditions of the solar atmosphere as described above, and adding a calculated contribution for the electrons. The result is

$$\beta(\text{Sun}) = 1{\cdot}7 \times 10^{-3}\,\text{cm}^{-1}.$$

If the measured shift is taken for β(helium), then a much lower value for β(Sun) is obtained. In any event, the collision shift for Ca 4227 A is a negligible component of the solar red shift, which is of order 5×10^{-2} cm^{-1}, and cannot account for the difference between observed and predicted solar red shifts.

Acknowledgments.—The work described in this paper would have been impossible but for the excellent performance of the Babcock grating, and the author wishes to express his gratitude to Dr Babcock for making available to the Observatory one of his gratings. The author also wishes to thank Professor H. H. Plaskett, F.R.S., for his constant assistance and advice during the course of these experiments.

University Observatory,
Oxford:
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References

(1) Adam, M. G., *M.N.*, **112,** 546, 1952.
(2) Bates, D. R., and Damgaard, A., *Phil. Trans.* A, **242,** 101, 1949.
(3) Bray, R. J., *M.N.*, **114,** 540, 1954.
(4) Foley, H. M., *Phys. Rev.*, **69,** 616, 1946.
(5) van de Hulst, H. C. and Reesinck, J. J. M., *Ap. J.*, **106,** 121, 1947.
(6) Kleeman, B., and Lindholm, E., *Ark. f. Mat. Astron. Fys.*, **32B,** No. 10, 1945.
(7) Kusch, H. J., *Zs. f. Astrophys.*, **45,** 1, 1958.
(8) Lang, K., *Acta Phys. Austriaca*, **11,** 76, 1957.
(9) Lindholm, E., *Ark. f. Mat. Astron. Fys.*, **32A,** No. 17, 1945.
(10) Lindholm, E., Über die Verbreiterung und Verschiebung von Spektrallinien, Dissertation, Uppsala, 1942.
(11) Pierce, A. K., *J. Op. Soc. Am.*, **47,** 6, 1957.
(12) Plaskett, H. H., *M.N.*, **95,** 160, 1934.
(13) Spitzer, L., *M.N.*, **110,** 216, 1950.
(14) Trefftz, E., *Zs. f. Astrophys.*, **29,** 287, 1951.
(15) Unsöld, A., *Physik der Sternatmosphären*, 2nd. edn., Ch. 11, Springer, 1955.

COLLISION SHIFTS IN THE ARGON SPECTRUM

W. R. Hindmarsh and K. A. Thomas

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Summary

The collision shifts of two argon lines, λ 7067 A and λ 6965 A, have been measured, the lines being excited in a D.C. glow discharge in pure argon. The lines were examined by means of a Fabry–Perot interferometer crossed with a Hilger prism spectrograph. The interference ring diameters were measured for a variety of argon pressures and discharge currents. It is shown that the measured shifts were due to collisions between two argon atoms, and not to the influence of charged particles. The shift for λ 7067 is $-0{\cdot}61 \pm 0{\cdot}05 \times 10^{-20}$ cm^{-1} per atom per cc of argon, and for λ 6965 is $-0{\cdot}65 \pm 0{\cdot}05 \times 10^{-20}$ cm^{-1} per atom per cc of argon, where the negative sign indicates a red shift. These results are shown to imply that the short range repulsive forces between argon atoms can be of little importance in producing the shifts, in contrast to the case of the calcium line λ 4227 A perturbed by helium.

Introduction.—One of the two possible methods of investigating collision effects in atomic spectra discussed in the preceding paper is to excite the spectrum in a gaseous discharge, and examine the spectral lines with the use of an interferometer. Such a method is the only one available for a spectrum of high excitation potential, as is the argon spectrum, where thermal excitation is impossible. If the results obtained in such measurements are to be physically interpretable, it is necessary to establish whether collision effects are due to neutral particles, electrons or ions, or all three. This paper describes the measurement and interpretation of collision shifts for the two infra red argon lines λ 7067 A and λ 6965 A at pressures of argon below 3 cms mercury. The transitions giving rise to the lines are

$$3p\ 4s(4s\ 1^{\circ}) - 3p\ 4p(4p\ 8),\ \lambda\ 7067$$

and

$$3p\ 4s(4s\ 1^{\circ}) - 3p\ 4p(4p\ 9),\ \lambda\ 6965,$$

where the notation is that of Revised Multiplet Tables (3). These lines should not suffer from " resonance " broadening to any appreciable extent, but, since the lower level is metastable, they may suffer from self absorption broadening. Neither resonance nor self absorption broadening can produce any shift in the conditions of this experiment.

Experimental method.—The discharge tube is shown in Fig. 1. It was made of ' Pyrex ' glass, and was 1 cm in diameter. The anode was a cylindrical copper insert (C) shaped to fit the ground glass sockets (G). Sealing wax was used to make the joints vacuum tight. The cathode consisted of a copper block (B) inserted into the copper-glass seal (S). The window (W), attached to the tube by means of black wax, allowed light from only the positive column

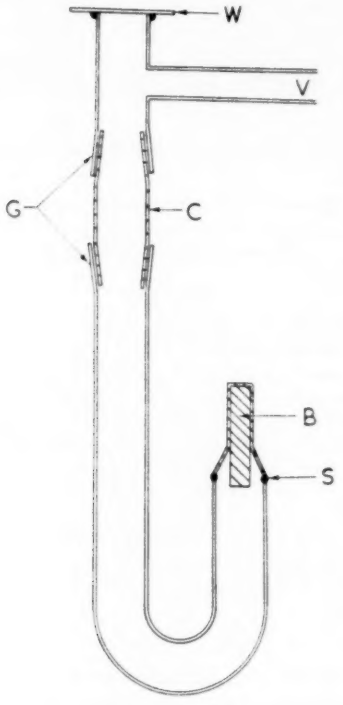

FIG. 1.

of the discharge to emerge. The light from the cathode region, where the macroscopic electric fields are high, did not enter the optical system. The discharge tube was cooled by means of a water bath maintained at about 12 °C.

The discharge tube was attached to a vacuum line, and the argon pressures measured by means of a manometer containing diffusion pump oil of density 1·09 gm cm^{-3}. A liquid oxygen cooled trap lay between the manometer and the discharge tube.

The voltage developed across the discharge was about 300 volts, and was almost independent of current and pressure in the range 50–150 mA and 0·8–23·5 mm of mercury.

Light from the positive column was passed through the interferometer in a collimated beam, and the interference fringes were focused on the slit of a Hilger medium spectrograph with glass optics by means of a Zeiss achromat of focal length 84 cm. Light from a G.E.C. cadmium lamp could also be passed through the interferometer, and the constancy of the interference-ring diameters of the cadmium red line λ 6438 used as a check on the stability of the interferometer.

The interferometer plates were 6 cm in diameter and made of fused quartz. Silver films of estimated reflectivity 87 per cent and transmissivity 11 per cent at 7000 A were deposited on them by evaporation. These gave a theoretical instrumental half-intensity width of 0·005 cm^{-1} with a 3 cm spacing between

the plates, and 0·004 cm^{-1} with a 4 cm spacing, compared with the Doppler width of the argon lines at 300 °K of 0·03 cm^{-1}. The room containing the interferometer and spectrograph was temperature controlled. The interferometer was enclosed in an air-tight box to maintain the air density constant. Measurements were made with 3 and 4 cm spacers in the interferometer.

Four exposures were made on each plate. First the cadmium spectrum was exposed. Secondly the argon spectrum was photographed at a certain discharge current and argon pressure. Thirdly the argon spectrum was again photographed but at either a different current or a different pressure. Fourthly the cadmium spectrum was exposed again. Three argon pressures were used, namely 0·8 mm, 12·0 mm and 23·5 mm of mercury, and three argon discharge currents, namely 50 mA, 100 mA, and 150 mA. Kodak I.R.E.R. plates were used, and exposure times for the argon lines were between 10 and 60 min, while for the cadmium red line the time was 3 min.

Measurement of the shift.—On each plate the interference ring diameters of the two exposures of Cd 6438 were measured. If the interferometer displacement revealed by differences in the ring diameters between the two exposures amounted to more than 1/200 of an order of interference, the plate was rejected. On the plates which survived this test, the ring diameters of the two argon lines were measured for each argon exposure. These measured diameters were corrected to allow for the very small displacement of the etalon during the argon exposures by assuming that the displacement expressed as a fraction of an order of interference deduced from the cadmium ring-diameters proceeded at a uniform rate. This correction was always much smaller than the measured argon shifts. The change in wavelength of the argon lines between two exposures on the same plate was then deduced from the corrected ring diameters by the method of rectangular array. When the two argon exposures concerned were on different plates, the shift between the corresponding cadmium exposures was subtracted from the corrected shift between the argon exposures to give the true shift of the argon lines. This technique of measuring shifts is similar to that used in the measurement of isotope shifts in atomic spectra by Pery (**2**).

Table I shows the results for changes of discharge currents at a fixed pressure, while Table II gives the shifts measured for changes of pressure at a fixed current. These tables show clearly that there is no measurable change of wavelength with discharge current, and that there are no significant differences between the results for different interferometer spacers. The error quoted is in all cases the standard deviation of the mean.

Discussion of results.—The measured shifts may be due to changes in the static electric fields in the discharge, collisions between charged particles and argon atoms, or collisions between argon atoms.

The estimated theoretical Stark effect of both lines is about 10^{-4} cm^{-1} for an electric field of 1 kV/cm, and dependence on the field strength is quadratic in form. It is most improbable that fields of this order can exist in the positive column of a glow discharge, except in the space charge sheath formed near the walls. Since this sheath extends into the tube only for a distance of one mean free path, the light from it cannot make any observable contribution to the measured shifts. It is therefore concluded that Stark effects caused by the macroscopic electric fields of the discharge cannot be responsible for the observed shifts.

TABLE I

Change of wave number with current

Wavelength	Pressure mm Hg	First and second currents mA	Shift with 3 cm spacer (no. of plates) 10^{-3} cm^{-1}	Shift with 4 cm spacer (no. of plates) 10^{-3} cm^{-1}	Mean 10^{-3} cm^{-1}
	0·8	100; 150	+0·1 ± 0·4 (3)	−0·1 ± 0·1 (3)	0·0 ± 0·2
		50; 150	+0·2 ± 0·3 (3)	0·0 ± 0·4 (3)	+0·1 ± 0·2
	12·0	100; 150	+0·1 ± 0·1 (3)	−0·1 ± 0·1 (3)	0·0 ± 0·1
6965		50; 150	+0·2 ± 0·2 (3)	−0·3 ± 0·2 (3)	0·0 ± 0·2
	23·5	100; 150	+0·3 ± 0·1 (3)	−0·4 ± 0·4 (3)	0·0 ± 0·2
		50; 150	+0·3 ± 0·3 (3)	+0·2 ± 0·1 (2)	+0·2 ± 0·2
	0·8	100; 150	+0·1 ± 0·3 (3)	−0·1 ± 0·5 (3)	0·0 ± 0·3
		50; 150	0·0 ± 0·3 (3)	0·0 ± 0·4 (3)	0·0 ± 0·3
	12·0	100; 150	+0·2 ± 0·2 (3)	−0·7 ± 0·3 (3)	−0·3 ± 0·2
7067		50; 150	−0·3 ± 0·2 (3)	−0·3 ± 0·1 (3)	−0·3 ± 0·1
	23·5	100; 150	+0·1 ± 0·3 (3)	−1·0 ± 0·1 (3)	−0·5 ± 0·3
		50; 150	+0·2 ± 0·4 (3)	+0·1 ± 0·4 (2)	+0·2 ± 0·3

TABLE II

Change of wave number with pressure (average of plates at 50, 100 *and* 150 mA)

Wavelength	First and second pressures mm Hg	Shift with 3 cm spacer (no. of plates) 10^{-3} cm^{-1}	Shift with 4 cm spacer (no. of plates) 10^{-3} cm^{-1}	Mean 10^{-3} cm^{-1}
6965	0·8; 23·5	−4·6 ± 0·1 (6)	−5·4 ± 0·5 (5)	−5·0 ± 0·3
	0·8; 12·0	−2·4 ± 0·2 (6)	−2·3 ± 0·2 (6)	−2·4 ± 0·2
7067	0·8; 23·5	−4·1 ± 0·2 (5)	−5·2 ± 0·4 (4)	−4·7 ± 0·3
	0·8; 12·0	−2·4 ± 0·3 (6)	−2·2 ± 0·3 (6)	−2·3 ± 0·2

$1{\cdot}0 \times 10^{-3}$ cm$^{-1} \equiv 5{\cdot}0 \times 10^{-4}$ A at $\lambda = 7000$ A.

Assuming the same value for the Stark coefficient of the argon lines, we find that Lindholm's theory for broadening by electron impact predicts that an electron density of 10^{12} electrons per cc is required to produce a line shift of 10^{-4} cm^{-1}, if the electron temperature is taken as 3×10^4 °K. The value of the shift is very insensitive to the temperature assumed. Estimates based on measurements of the electron drift velocity in an argon discharge by Nielsen (**1**) suggest that the highest electron density likely to be encountered in our experiments is 5×10^{11} electrons per cc, so that the shifts due to electron impact are negligible. Effects due to ions will be very much smaller. (See, for example, Unsöld (**4**)).

These theoretical estimates are supported by our observation that there is no detectable shift due to current changes. The conclusion is therefore drawn that the measured shifts are due to collisions between two argon atoms. The resonance forces cannot give rise to a shift, so that the van der Waals attractive and short-range repulsive forces must be responsible. As the theory predicts, the observed shifts are proportional to the pressure change at a fixed temperature.

The final result for the shift is

λ 7067 $\beta/N = -0{\cdot}61 \pm 0{\cdot}05 \times 10^{-20}$ cm^{-1} per atom per cc of argon

λ 6965 $\beta/N = -0{\cdot}65 \pm 0{\cdot}05 \times 10^{-20}$ cm^{-1} per atom per cc of argon.

These values of β/N are substantially larger than that found for Ca 4227 perturbed by helium, and, moreover, are in the expected direction, towards the red. Assuming that Lindholm's theory is correct for the shift, and that the interatomic forces have a potential varying as the inverse sixth power of the distance between two argon atoms, we find that the collision radius necessary to give these shifts is 4×10^{-8} cm. If the short-range repulsive forces are important, an even larger collision radius would be necessary to give the observed shift, since the repulsive forces tend to produce a violet shift. Since the radius of an argon atom in the ground state is only $1{\cdot}56 \times 10^{-8}$ cm, it seems probable that the short-range forces are not, in fact, very important in this case, in contrast to what was found for Ca 4227 in the preceding paper (**5**).

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University Observatory,
Oxford:
1958 *November* 20.

References

(1) Nielsen, R. A., *Phys. Rev.*, **50**, 950, 1936.
(2) Pery, A., *Proc. Phys. Soc.* A, **67**, 181, 1954.
(3) Russell, H. N. and Moore, C. E., *Revised Multiplet Table*, Contributions from Princeton University Observatory, No. 10, 1945.
(4) Unsöld, A., *Physik der Sternatmosphären*, 2nd edn., Ch. 11, Springer, 1955.
(5) Hindmarsh, W. R., *M.N.*, **119**, 11, 1959.

ANGULAR DIAMETER MEASUREMENTS OF THE RADIO SOURCES CYGNUS (19N4A) AND CASSIOPEIA (23N5A) ON A WAVELENGTH OF 10·7 CM

B. Rowson

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Summary

The apparatus used in measuring the angular diameters of radio sources at a wavelength of 10·7 cm is described. Results for two directions at right angles are given for both Cassiopeia (23N5A) and Cygnus (19N4A). These show that the former is circularly symmetrical and much the same size as it is at metre wavelengths, while the latter is markedly asymmetrical with a major axis that may be slightly larger at the higher frequency. The position angle of this major axis has been found to be $109° \pm 2°$.

Introduction.—The apparent angular diameters of the most intense radio sources have already been measured successfully at metre wavelengths (Smith 1952, Mills 1952, 1953, Hanbury Brown *et al.* 1952, Jennison and Das Gupta 1956). Subsequently the diameters of some of these sources have been measured at 500 Mc/s, (Conway 1956) and at 2800 Mc/s, (Rowson 1957). In both of these papers the high frequency results have been compared with the metre wave results and no very significant differences found. The present paper describes the apparatus used at 2800 Mc/s and gives the results obtained for Cygnus (19N4A) (R.A. $19^h\ 57^m\ 45^s$, Dec. $40°35'$) and Cassiopeia (23N5A) R.A. $23^h\ 21^m\ 12^s$, Dec. $58°\ 32'$) for two directions at right angles.

Apparatus.—The apparatus consisted of a two aerial interferometer working on a wavelength of 10·7 cm. Each aerial was a focal plane paraboloidal reflector illuminated by a dipole primary feed (Fry and Goward 1950) coupled to a waveguide which ran along the axis of the paraboloid to a crystal mixer behind the reflecting surface. The larger reflector was 25 ft in diameter and consisted of half-inch wire mesh supported on a tubular iron frame giving a beam width of about 1° between half power points. The 10 ft reflector was made of sheet iron and gave a beam width of $2\frac{1}{2}°$. The aerials were supported on altitude-azimuth mountings, one being fixed to a concrete block while the other was mounted on a trailer, so that the length and direction of the baseline could be changed.

A waveguide mixer and intermediate frequency preamplifier were mounted in a box behind each aerial, and the signal at intermediate frequency was fed along equal lengths of coaxial cable to the main amplifier. Here the signals were added together, further amplified, and the beat frequency between the two components obtained in a detector.

Separate klystron local oscillators were mounted on each aerial and about one per cent of the power from these oscillators was fed into the mixers via directional couplers. The rest of the power, which would normally be dissipated in a dummy load, was fed into long lengths of coaxial cable to a crystal mixer placed between the two aerials, for baselines shorter than 800 wavelengths. For longer baselines it was found impossible to get sufficient signal through the cables because of their very high attenuation at a frequency of 2800 Mc/s. Thus for these longer baselines, the local oscillator at the distant aerial was radiated from a 4 ft paraboloidal aerial and received on a similar aerial placed near the coaxial crystal mixer. This mixer produced the beat frequency between the two klystrons which was amplified by a selective amplifier at 20 kc/s and fed to a Foster–Seeley discriminator, the d.c. output of which was used to control the frequency of one of the oscillators to keep the beat frequency stable to within ± 1 kc/s (Fig. 1).

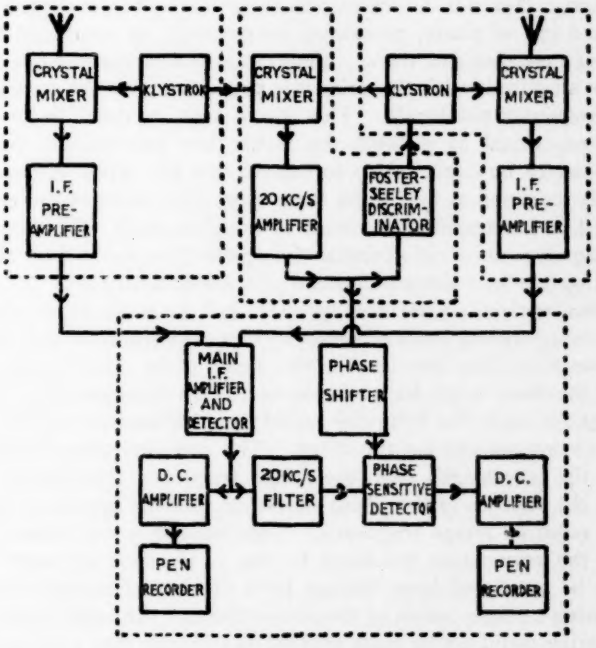

FIG. 1.—*Block diagram of apparatus.*

Interference fringes were obtained by selectively amplifying the intermediate frequency amplifier's video output and detecting it with a coherent detector which used as its reference wave the 20 kc/s beat frequency between the two klystrons. The signal from this detector was amplified in a d.c. amplifier and displayed on a pen recorder. The total power from the intermediate frequency amplifier was also recorded for gain calibration. The theory of an interferometer using local oscillators of slightly different frequencies in the two channels has been given by Hanbury Brown, Palmer and Thompson (1955).

When using East–West baselines longer than 400 wavelengths it was found that the fringe frequency was inconveniently high, while for observations at transit with North–South baselines the fringe frequency was zero. To overcome these difficulties a system known as the 'phase shifter' was developed, which changed the frequency of the reference signal to the coherent detector by a few cycles per minute. This consisted of a polyphase resistance capacity oscillator driving two balanced modulators in phase quadrature. The 20 kc/s signal was also fed to these two modulators in phase quadrature so that the sum of their outputs consisted of a single sideband differing from the original signal by the local oscillator frequency. This method was used in preference to a continuously rotating magslip because of the high frequency of the reference wave.

As image frequency rejection was not applied, signals were received in two bands on either side of the local oscillator frequency, the total separation between these two bands being twice the intermediate frequency. Thus, as the source moved through the aerial beams, two sets of fringes of slightly different frequency were produced. Hence if the source was observed for long enough these fringes moved in and out of phase, producing fringes with an amplitude modulation in the form of a rectified sine wave. Thus it can be seen that the rate of variation of the fringe amplitude with the delay was proportional to the separation of the two radio frequency passbands. This meant that, in order to keep the time delays as non-critical as possible the rather low intermediate frequency of 13·5 Mc/s had to be used. Also to ensure that the phase dispersion of the two channels remained as nearly the same as possible most of the amplification was obtained in an amplifier common to both channels. This resulted in a sacrifice in signal to noise ratio because the signals from the two aerials had to be multiplied together in a detector instead of a linear multiplier.

Operational method.—After the baseline had been changed, the movable aerial was levelled and the baseline surveyed with a theodolite, so that the variation of the difference in delay between the two signal paths could be calculated as a function of the hour angle for each source. This function was then used to find the length of cable that had to be added to one of the intermediate frequency cable links to compensate for the delay. The first derivative of this function divided by the wavelength gave the fringe frequency that would have been observed in the absence of the phase shifter, so that the apparatus could be set to give the required fringe frequency. This frequency was chosen to be low enough for the attenuation produced by the 12 sec time constant in the d.c. amplifier to be small and large enough for a number of complete cycles to be obtained during a single transit of the source through the aerial beams.

As the aerials could not be made to track on a source, they were set for a given hour angle and the source allowed to drift through the beams. By this method it was found possible to make a number of observations near to meridian transit, usually at intervals of twenty minutes.

Since the beamwidth of the larger aerial only amounted to one degree and as the delay had to be set to the nearest metre, the effect of varying the altitude and azimuth of the aerials and the length of the delay cable from the calculated optimum values was investigated experimentally for each baseline.

As far as total power was concerned the apparatus consisted of two straight receivers with their outputs added together. Thus it was not possible to record the changes in total power due to the source passing through the aerial beams,

because of apparatus instability. This meant that the absolute fringe visibility could not be measured, and so fringe amplitudes were measured, corrections were applied for gain changes in the equipment and then the results were normalized to give unit fringe visibility at zero baseline.

TABLE I

Cassiopeia (23N5A)

E. W. Baselines		N. S. Baselines	
Length of baseline in wavelengths	Normalized fringe amplitude	Length of baseline in wavelengths	Normalized fringe amplitude
120	0·98 ± 0·05	136	0·89 ± 0·05
142	0·99	430	0·77
400	0·63	726	0·27
550	0·47	880	0·06
700	0·32	1730	0·08
894	< 0·10		
1290	0·13		
1750	< 0·10		

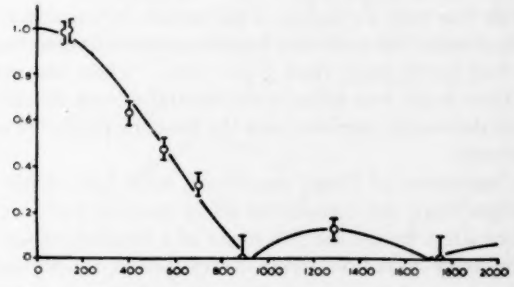

FIG. 2.—*Cassiopeia* (23N5A) *East–West. Abscissa : length of baseline in wavelengths. Ordinate : normalized fringe amplitude.*

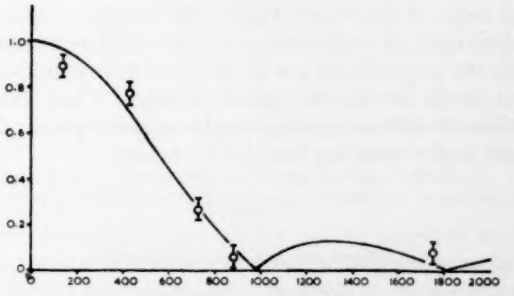

FIG. 3.—*Cassiopeia* (23N5A) *North–South. Abscissa : length of baseline in wavelengths. Ordinate : normalized fringe amplitude.*

The Cassiopeia (23N5A) *source.*—Measurements of the fringe amplitude of the Cassiopeia source have been extended in the East–West direction from 700 wavelengths up to 1750 wavelengths. These measurements together with those published previously are shown in Table I and plotted in Fig. 2.

These confirm the original conclusion that the diameter of the source at this frequency is not larger, and not more than 25 per cent smaller, than at metre wavelengths. They also show that the second maximum in the Fourier transform is only 10 to 15 per cent of the primary maximum, which is taken to indicate that the source is devoid of very large scale fine structure such as is found in the Cygnus source.

The fringe visibility has also been measured with approximately North–South baselines of lengths up to 1750 wavelengths. The results of these measurements are shown in Fig. 3. It can be seen that the diameter here is the same as the East–West diameter to within the errors of measurement, thus showing that the source is nearly circular as it is at longer wavelengths. Here again there is no pronounced second maximum in the transform.

The Cygnus (19N4A) *source.*—As it was suspected that the Cygnus source might be markedly asymmetrical with its minor axis in an approximately North–South direction, measurements were taken first with North–South baselines in order to find the direction of the minor axis accurately. This was done by observing the source for several hours on either side of upper culmination to measure the variation of fringe amplitude with hour angle.

As the source was only 12° south of the zenith at transit and moved so as to describe a circle around the pole star, baseline foreshortening was a maximum at transit and so was never more than 3 per cent. Thus any variation of fringe visibility with hour angle was either instrumental or was due to variations in the angle θ, between the source meridian and the baseline projected on to the celestial sphere at the source.

Systematic variations of fringe amplitude with hour angle due to receiver sensitivity changes were not considered likely because each set of observations was taken within a few hours and the mean of a number of sets of observations obtained, while delay variations were compensated for by adding calculated lengths of cable to one channel for each individual transit of the source through the aerial beams. The attenuation produced by this cable was allowed for numerically.

As the hour angle of the source varied, the elevation changed and, in view of the short wavelength, it was possible that the aerial gains might change. As total power from the source could not be recorded from either aerial this possible systematic error could not be eliminated directly. Thus measurements were made with baselines at different position angles so that a given value of θ occurred for different hour angles from one baseline to the next.

TABLE II

Minor axis of Cygnus (19N4A)

Length of baseline in wavelengths	Normalized max. fringe amplitude
136	0·99 ± 0·10
430	1·00
726	0·88
1730	0·97
2480	0·82
3380	0·75

Figure 4 shows the fringe visibility plotted against hour angle and values of θ for three different baselines. It can be seen that the primary maximum occurs at a constant value of θ but at different hour angles for each baseline. Also it is evident that the fringe amplitude changes more rapidly as the length of the baseline is increased. As the maximum fringe amplitude occurs when the minor axis is in the same direction as the projected baseline, the position angle of this axis is $19 \pm 2°$, so that of the major axis is $109° \pm 2°$.

It is interesting to note that the results for the longest baseline have a second maximum. This can readily be explained in terms of a source model consisting of two centres of emission similar to that suggested by Jennison and Das Gupta (1953). The curves drawn in Figs. 4, 5 and 6 are the calculated fringe visibility for such a model.

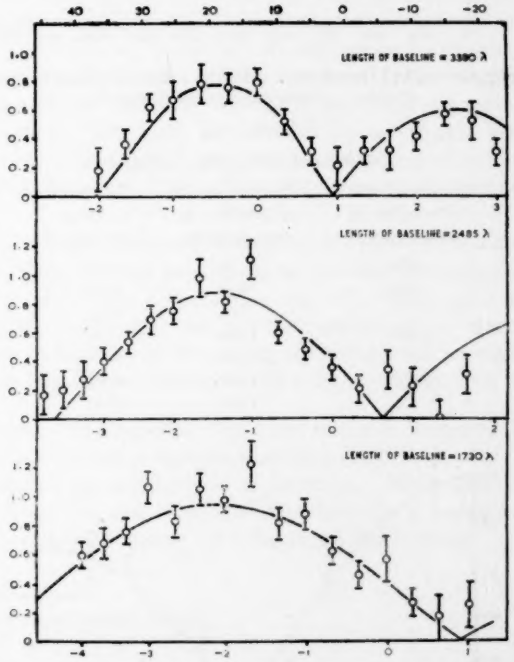

FIG. 4.—*Position angle of the minor axis of Cygnus* (19N4A) *Abscissae : top, θ in degrees common to all diagrams ; others, hour angle in hours. Ordinates : normalized fringe amplitude.*

In Fig. 5 the maximum fringe visibility against length of baseline is plotted. This shows that the fringe visibility is only just beginning to fall at the longest baseline, from which it is estimated that the width of the minor axis is only about 20″.

In obtaining the Fourier transform of the intensity distribution along the major axis a difficulty was encountered in that the ground was not available for baselines of the necessary length in the required position angle. Thus measurements were made with baselines in position angles of approximately 80° and observations were in general taken several hours before transit. The effective length of baseline for each observation was then calculated by multiplying the

actual length by the sine of the angle between the aerial beams and the baseline to correct for foreshortening, and by the cosine of the angle between the major axis of the source and the baseline projected on to the celestial sphere to allow for

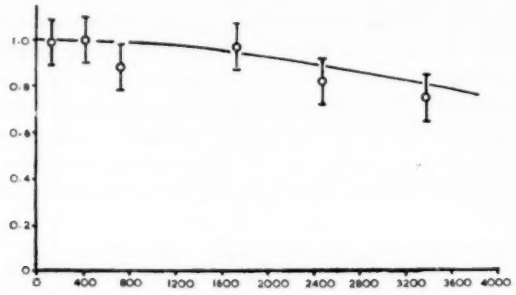

FIG. 5.—*Cygnus* (19N4A) *minor axis. Abscissa : length of baseline in wavelengths. Ordinate : normalized fringe amplitude.*

TABLE III

Major axis of Cygnus (19N4A)

Actual length of baseline in wavelengths	Effective length of baseline in wavelengths	Normalized fringe amplitude
180	180	$0{\cdot}96 \pm 0{\cdot}10$
890	630	0·57
	800	0·30
1300	1020	$<0{\cdot}15$
	1180	0·24
1750	1500	0·68
	1670	0·67

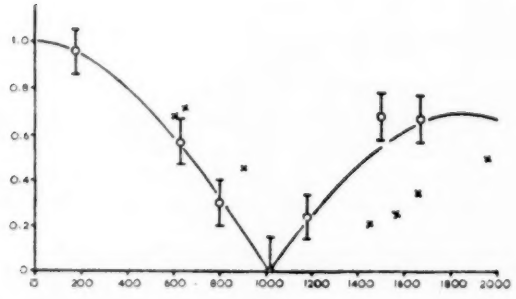

FIG. 6.—*Cygnus* (19N4A) *major axis.* × *Jennison and Das Gupta.* ○ 10 *cm results. Abscissa : length of baseline in wavelengths. Ordinate : normalized fringe amplitude.*

loss of resolution due to the projected baseline not being parallel to the major axis of the source. This second correction was considered justified since it had already been shown that the lengths of the baselines used in the East–West direction would be insufficient to produce significant resolution in the direction of the minor axis. When these corrections were applied, it was found that the maximum resolution was obtained shortly after the angle between the projected baseline and the major axis of the source had passed through zero, since here the increasing angle offset the increasing resolution due to decrease of baseline foreshortening.

In Table III two values of fringe amplitude are given for each of the longer East–West baselines. One corresponds to the maximum resolution obtainable, while the other is the mean of the results taken on either side of the maximum. Figure 6 shows these results plotted against the effective length of baseline. For comparison the results given by Jennison and Das Gupta (1956) for baselines with position angle 113° are plotted on this diagram. It is seen that the first minimum in the Fourier transform occurs at appreciably shorter baselines and that the second maximum is noticeably larger at the higher frequency. If these results are interpreted as indicating a source with two separate centres of emission, this would mean that the two parts of the source, while remaining much the same size, had increased their angular separation by about 20 per cent.

Discussion.—These results suggest that the size of the Cygnus source has increased in going from a frequency of 125 Mc/s to 2800 Mc/s while Conway (1956) comes to the conclusion that no appreciable change takes place in going from 125 Mc/s to 500 Mc/s. However, on examining his graph for the Cygnus source it is found that he has plotted the results of Jennison and Das Gupta as though they were fringe visibility whereas in fact they are equal to the square of the fringe visibility (Hanbury Brown and Twiss 1954). When the square root of Jennison and Das Gupta's results are plotted together with those due to Conway it is again found that the Cygnus source appears larger at the higher frequency.

Taking the major axis of the source to be in the East–West direction, Jennison and Das Gupta (1953) found reasonable agreement between their results and those of Smith. However, if these two sets of results are compared on the assumption that the major axis has position angle 109° at metre wavelengths, there is again an indication of increasing diameter with increasing frequency, although here the two frequencies differ by a factor of less than two.

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University of Manchester,
Jodrell Bank Experimental Station,
Cheshire:
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References

Conway, R. G., 1956, *Observatory*, **76**, 235.
Fry, D. W. and Goward, F. K., *Aerials for Centimetre Wavelengths* (Cambridge University Press, 1950), page 55.
Hanbury Brown, R., Jennison, R. C. and Das Gupta, M. K., 1952, *Nature* (*Lond.*), **170**, 1061.
Hanbury Brown, R., Palmer, H. and Thompson, A. R., 1955, *Phil. Mag.*, **46**, 857.
Hanbury Brown, R. and Twiss, R. Q., 1954, *Phil. Mag.*, **45**, 663.
Jennison, R. C. and Das Gupta, M. K., 1956, *Phil. Mag.* (Ser. 8), **1**, 55 ; 1953, *Nature* (*Lond.*), **172**, 996.
Mills, B. Y., 1952, *Nature* (*Lond.*), **170**, 1063 ; 1953, *Aust. J. Phys.*, **6**, 452.
Rowson, B., 1957, *Proc. Phys. Soc.* B, **70**, 328.
Smith, F. G., 1952, *Nature* (*Lond.*), **170**, 1065.

VELOCITY ELLIPSOIDS AND THE GRAVITATIONAL POTENTIAL OF THE GALAXY

P. A. Wayman

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Summary

In 1956 a model of the gravitational potential of the Galaxy was derived by M. Schmidt from an analysis of the circular velocities indicated by the 21 cm radiation from interstellar material and of other data. It is found that, with a little modification, Schmidt's model can be regarded as an example of a type of steady-state galaxy described in 1941 by Camm. The velocity ellipsoid at the Sun's radius in this type of galaxy may have three unequal axes and it is possible to discuss the variation of these axes in different parts of the Galaxy. One result is a tilt of the " direction of star-streaming " a fairly short distance above or below the galactic plane.

1. *Introduction.*—Certain features of the Galaxy, such as the presence of dissolving clusters and the location of certain types of stars in spiral arms, deny the possibility of representing the Galaxy entirely as a system of stars in a steady state, and yet the age of certain other members indicates that the general shape and size of the Galaxy is unlikely to be changing rapidly. Oort (**1**) has thus envisaged the formation of a disk-structure as representing a quasi-stationary stage in the condensation of the interstellar material towards the centre of the Galaxy wherein further collapse is offset by the galactic rotation; groups of interstellar material have interacted among themselves until they all partake in rotation about an axis which was originally the axis of the total angular momentum of the material concerned, a disk-structure having minimized this interaction. Conversion of this material into the older background stars of the galactic disk, following the formation and subsequent dissolution of clusters, may be expected to produce amongst these stars some features associated with a steady state in their distribution and their velocities even though the relaxation time of their own mutual interaction may itself be far too great to have caused such features. At the same time it may also be possible to trace the remnants of group motions among the same stars, as has recently been re-emphasized by Eggen (**2**). In any case it is important to know what features of the Galaxy are, and what features are not, those which might be found in a steady-state galaxy. It is in this spirit that we consider here some features in the relation between the distribution of stellar density and the observed motions of stars in the region near the Sun, taking into account what is known of the gravitational potential of the Galaxy as a whole.

For this purpose a suitable model of the distribution of mass in the Galaxy is one with rotational symmetry about an axis and with a plane of symmetry, the galactic plane, perpendicular to this axis. The representation of stellar motions by the ellipsoidal hypothesis of Schwarzschild (**3**) was originally empirical, but it was soon realized that it offered some chance of establishing possible relations between stellar motions and stellar density. Among the first attempts to do this

was that of Jeans (**4**) in 1915. One of Jeans' results was that in a system with axial symmetry a non-isotropic distribution of velocities was only possible, for a steady state, if the distribution of velocities at a given point, or in a small region, had symmetry about an axis perpendicular to the plane through the point considered and containing the axis of symmetry of the system as a whole. That is, star-streaming, if it exists, must be in a "transverse" direction. Reference to this result has frequently been made, up to the present time, (see, e.g., Smart (**5**), Oort (**6**), Schmidt (**7**), Woolley (**8**)), even though other possibilities have been discussed, first by Eddington (**9**), writing simultaneously with Jeans, and later by Clark (**10**), Chandrasekhar (**11**), Camm (**12**) and Parenago (**13**). It was curious that Eddington's result predicted the existence of *radial* star-streaming for the case of axial symmetry and some time later Eddington himself (**14**) thought that between them he and Jeans had shown that neither radial nor transverse star-streaming were possible in the axially-symmetric steady-state galaxy. Chandrasekhar (**15**), however, showed that Eddington's formulation, though valid, was not general; it had seemed too restrictive by the forms that it imposed on the gravitational potential function and Chandrasekhar pointed out that Eddington's solution involved the unnecessary assumption that a function of three independent variables would be a perfect differential.

In the meantime the study of motions within the galactic plane based upon the assumptions of a steady state had been shown by Oort (**6**) and Lindblad (**16**) to yield important results connected with the newly-discovered existence of galactic rotation about a distant centre. These results were:

(i) a striking interpretation of Stromberg's empirical parabolic relation (**17**) between velocity dispersion and mean motion for different groups of objects in the Galaxy;

(ii) an interpretation of the strong asymmetry existing for the velocities of the high-velocity stars; and

(iii) a relation between Oort's constants of galactic rotation, A and B, and the ratio of axes of the velocity ellipse in the galactic plane.

Besides these important and well-known developments, Oort (**18**) had carried out an investigation of stellar motions in a direction perpendicular to the galactic plane based upon the assumption of a steady state existing in the density and velocity distributions in this direction for the region at the Sun's distance from the galactic centre.

It became generally accepted that Jeans' result concerning the equality of the "radial" and "z" axes of the velocity ellipsoid was inevitable under the assumption of a steady state unless the potential function could be separated. That is, unless $\partial^2\Phi/\partial R\partial z = 0$. This condition would hold at the galactic plane but not over an extended region. A more complete picture of the consequences of a steady-state hypothesis was given by Camm (**12**) and it is of interest to know how far his results can be applied to the information on the density distribution in the Galaxy which has been provided by the interpretation of the 21 cm radiation profiles in terms of circular velocity $\omega_c(R)$. Kwee, Muller and Westerhout (**19**) published a discussion of such profiles and Schmidt (**7**) utilized these data to the full in constructing a potential function based on a model consisting of a series of spheroidal mass distributions. He also made use of information on the potential gradient in the z-direction at the Sun's distance from the galactic centre. Schmidt's model seems to be restricted by having to

assume uniform eccentricity inside his elemental spheroidal mass-distributions. This limitation shows itself in giving what is perhaps a too great degree of flattening for the central regions of the Galaxy and possibly leads to the postulation of a considerable amount of unidentified matter contributing to the mass of the Galaxy. Nevertheless Schmidt's Table 7 seems to provide the best available numerical values of the potential function if we bear in mind these limitations, as well as those which stem from the uncertainties in the basic data; for example, in the value assumed for the Sun's distance from the galactic centre, 8·2 kpc.

2. *Steady-state theory in cylindrical coordinates.*—If (q_1, q_2, q_3) are generalized coordinates of position with canonical momentum coordinates (p_1, p_2, p_3) and there exists a frequency function for the distribution of stars in position and velocity

$$dN = \Psi'(q_1, q_2, q_3, p_1, p_2, p_3)\, dq_1\, dq_2\, dq_3\, dp_1\, dp_2\, dp_3$$

Liouville's theorem, or the equation of continuity, applied to the six-dimensional phase-space, states that Ψ' must satisfy the equation

$$\frac{\partial \Psi'}{\partial t} + \sum_s \left\{ \frac{\partial H}{\partial p_s}\frac{\partial \Psi'}{\partial q_s} - \frac{\partial H}{\partial q_s}\frac{\partial \Psi'}{\partial p_s} \right\} = 0 \qquad (1)$$

where H is the Hamiltonian function per unit mass and the p's are generalized momenta per unit mass.

$$H = T(p_s, q_s) + \Phi(q_s) \qquad (2)$$

where T is the kinetic energy per unit mass and Φ the negative potential energy per unit mass.

It was shown quite generally by Camm (**12**) and Chandrasekhar (**20**) that for stellar systems which are in a steady state and which are of finite spatial extent it is only possible to have differential motions if the potential is characterized by axial symmetry. We consider this to be a suitable representation of the Galaxy and we choose cylindrical coordinates

$$q_1 = R,\ q_2 = \theta,\ q_3 = z.$$

The galactic centre (Fig. 1) is C, taken to be a distance $R_0 = 8{\cdot}2$ kpc from the Sun in a direction which has galactic coordinates $l = 327^\circ{\cdot}5$, $b = 0^\circ$.

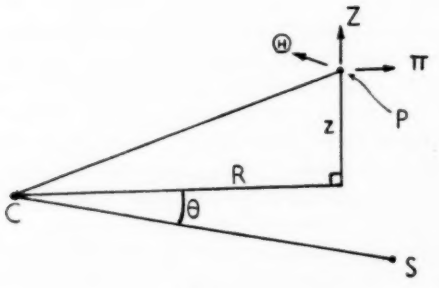

FIG. 1.

Axial symmetry in Φ implies $\partial\Phi/\partial\theta = 0$ and if, in addition, the plane $z = 0$ is a plane of symmetry, $\Phi = \Phi(R^2, z^2)$. The kinetic energy per unit mass is

$T=\frac{1}{2}(\Pi^2+\Theta^2+Z^2)$ where Π, Θ, Z are velocities at P (Fig. 1) in the "radial", "transverse", and "z" directions respectively. In terms of q_1, q_2, q_3,

$$T=\tfrac{1}{2}\,(\dot{q}_1^2+q_1^2\dot{q}_2^2+\dot{q}_3^2)$$

Then

$$\left.\begin{aligned} p_1&=\frac{\partial T}{\partial \dot{q}_1}=\dot{q}_1=\Pi \\ p_2&=\frac{\partial T}{\partial \dot{q}_2}=q_1^2\dot{q}_2=R\Theta \\ p_3&=\frac{\partial T}{\partial \dot{q}_3}=\dot{q}_3=Z \end{aligned}\right\} \tag{3}$$

and

$$H=\frac{1}{2}\left(p_1^2+\frac{p_2^2}{q_1^2}+p_3^2\right)+\Phi(q_1, q_3).$$

If we introduce the observed frequency distribution of velocities Ψ where

$$\Psi(R, \theta, z, \Pi, \Theta, Z)\; R\,dR\,d\theta\,dz\,d\Pi\,d\Theta\,dZ$$
$$=\Psi'\;(q_1, q_2, q_3, p_1, p_2, p_3)\,dp_1\,dp_2\,dp_3\,dq_1\,dq_2\,dq_3$$

we have

$$\frac{\partial \Psi}{\partial R}=\frac{\partial \Psi'}{\partial q_1}+\frac{\partial \Psi'}{\partial p_2}\frac{\partial p_2}{\partial R}=\frac{\partial \Psi''}{\partial q_1}+\Theta\frac{\partial \Psi''}{\partial p_2}$$

and

$$\frac{\partial \Psi}{\partial \Theta}=\frac{\partial \Psi'}{\partial p_2}\frac{\partial p_2}{\partial \Theta}=R\frac{\partial \Psi''}{\partial p_2}.$$

Liouville's theorem for Ψ under the steady-state condition $\frac{\partial \Psi}{\partial t}=0$, $\frac{\partial \Psi}{\partial \theta}=0$ becomes

$$\Pi\frac{\partial \Psi}{\partial R}+Z\frac{\partial \Psi}{\partial z}-\left(\frac{\partial \Phi}{\partial R}-\frac{\Theta^2}{R}\right)\frac{\partial \Psi}{\partial \Pi}-\frac{\Pi\Theta}{R}\frac{\partial \Psi}{\partial \Theta}-\frac{\partial \Phi}{\partial z}\frac{\partial \Psi}{\partial Z}=0. \tag{4}$$

Under the generalized ellipsoidal hypothesis of Chandrasekhar we postulate

$\Psi=\Psi(Q)$ where $Q=a\Pi^2+b\Theta^2-2b\Theta\Theta_0+cZ^2+2(f\Theta Z+g\Pi Z+h\Pi\Theta)+\sigma$.

The coefficients a, b, Θ_0, c, f, g, h and σ are functions of position only. We can write $d\Psi=\frac{d\Psi}{dQ}dQ$ and Q must satisfy (4) for all values of Π, Θ, and Z. The equation obtained by substituting Q in (4) in terms of a, b, etc., gives a polynomial of the third degree in Π, Θ, and Z. The coefficients of this polynomial can then be separately equated to zero, if this equation is an identity for all values of Π, Θ, Z. A number of equations that the coefficients a, b, etc., in Q must satisfy are obtained and we can take over Camm's result (**12**) for the solution of these equations for the present case. We must have:—

$$\left.\begin{aligned} a&=\alpha_0+\alpha_3z^2 & f&=0 \\ b&=\alpha_0+\alpha_3z^2+\alpha_2R^2 & g&=-\alpha_3Rz \\ c&=\alpha_4+\alpha_3R^2 & h&=0 \end{aligned}\right\} \tag{5}$$

$$\Theta_0=\frac{\alpha_1R}{b}=\frac{\alpha_1R}{(\alpha_0+\alpha_3z^2+\alpha_2R^2)} \tag{6}$$

$$\left.\begin{aligned} \frac{\partial \sigma}{\partial R}&=2a\frac{\partial \Phi}{\partial R}+2g\frac{\partial \Phi}{\partial z} \\ \frac{\partial \sigma}{\partial z}&=2g\frac{\partial \Phi}{\partial R}+2c\frac{\partial \Phi}{\partial z} \end{aligned}\right\} \tag{7}$$

where α_0, α_1, α_2, α_3, α_4 are constants for the system of stars being considered.

From (7), equating $\partial^2\sigma/\partial R\partial z$ in the two equations, it is necessary that Φ should satisfy the equation

$$\alpha_3\left\{3z\frac{\partial\Phi}{\partial R}-3R\frac{\partial\Phi}{\partial z}+(z^2-R^2)\frac{\partial^2\Phi}{\partial R\partial z}+Rz\left(\frac{\partial^2\Phi}{\partial R^2}-\frac{\partial^2\Phi}{\partial z^2}\right)\right\}+(\alpha_0-\alpha_4)\frac{\partial^2\Phi}{\partial R\partial z}=0. \tag{8}$$

This equation is equation 23 of Oort (**6**). He inserted the solution

$$\alpha_3=0,\ \alpha_0=\alpha_4$$

presumably on the basis that this differential equation is too restrictive on Φ otherwise. Camm showed, however, that if

$$\alpha_3\neq 0 \quad \text{and} \quad \alpha_0\neq\alpha_4$$

a solution exists of the form

$$\Phi=\{F(\xi)-G(\eta)\}/(\xi-\eta) \tag{9}$$

where (ξ, η) are elliptic-hyperbolic coordinates related to R and z by the equations

$$R^2=(\xi+\beta)(\eta+\beta)/2\beta \quad \text{and} \quad z^2=-(\xi-\beta)(\eta-\beta)/2\beta$$

the constant β being $\beta=(\alpha_0-\alpha_4)/2\alpha_3$.
Further, if $\gamma=\frac{1}{2}(\alpha_0+\alpha_4)$,

$$\{\tfrac{1}{2}\sigma+\gamma\Phi\}/\alpha_3=\{\xi G(\eta)-\eta F(\xi)\}/(\xi-\eta) \tag{10}$$

$F(\xi)$ and $G(\eta)$ are arbitrary single-valued functions of ξ and η.

For a group of stars governed by Ψ as a frequency-function of the form $\Psi=e^{-Q}$ the stellar density $\nu=\iiint_{\Pi,\Theta,Z}\Psi d\Pi\, d\Theta\, dZ$ is

$$\nu=\pi^{3/2}(abc)^{-1/2}\, e^{(-\sigma+b\Theta_0^2)}. \tag{11}$$

If we consider only the points $(R, 0)$ we can define the circular velocity $\Theta_c=R\omega_c$, given by $\Theta_c^2/R=(\partial\Phi/\partial R)_{R,0}$. At these points $g=0$, so that, using the first of equations (7), $\Theta_c^2=(R/2a)\partial\sigma/\partial R$. Also, $a=\alpha_0$, $b=\alpha_0+\alpha_2R^2$, $c=\alpha_4+\alpha_3R^2$ and $b\Theta_0^2=\alpha_1^2R^2/b$ so that

$$\frac{\partial}{\partial R}(b\Theta_0^2)=2\Theta_0^2\alpha_0/R.$$

Differentiating (11) w.r.t. R and inserting these expressions for a, b, c, and $\partial/\partial R(b\Theta_0^2)$ we easily obtain

$$\frac{1}{\nu}\frac{\partial\nu}{\partial R}=\frac{-\alpha_2R}{\alpha_0+\alpha_2R^2}-\frac{\alpha_3R}{\alpha_4+\alpha_3R^2}+\frac{2\alpha_0}{R}(\Theta_0^2-\Theta_c^2). \tag{12}$$

With a suitable change of notation this equation reduces to equation 31 of Oort (**6**).

3. *Schmidt's model of the Galaxy.*—The galactic model of Schmidt (**7**) is based upon the superposition of a number of non-homogeneous ellipsoids whose densities, varying in a particular way, are adjusted to fit primarily the circular velocities $\Theta_c(R)$ revealed in the inner parts of the Galaxy by the 21 cm radiation from interstellar hydrogen. For the outer parts an extrapolation was necessary. Existing information concerning the distribution of matter perpendicular to the galactic plane is also used. Schmidt finds that the superposition of four such non-homogeneous oblate spheroids gives a good approximation. The final solution employs the addition of a few homogeneous spheroids which are of quite small total mass. The final result is tabulated as $-\Phi$, $\partial\Phi/\partial R$ and $\partial\Phi/\partial z$

in Schmidt's Tables 7, 8, and 9 respectively. These data are used as the basis of the present investigation. Schmidt himself points out that his model suffers to a certain extent in having to assume uniform eccentricity inside his elemental spheroids, in that the resultant model is possibly too flattened towards the centre of the galactic system. However, lacking further information on the distant distribution of matter perpendicular to the galactic plane, no improvement seems possible. It is of interest in this respect that Parenago (**13**) suggested a form of Φ given by $\Phi=\Phi(R).\phi(z)$; he represented observed densities of RR Lyrae stars on the basis that $\Phi(0, z)/\Phi(0, 0)=\Phi(R_0, z)/\Phi(R_0, 0)$. There is however some uncertainty in his interpretation of the observations and Parenago's form of Φ is not in any way compatible with Schmidt's model. It implies that

$$\left(\frac{\partial^2\Phi}{\partial z^2}\right)_{R,0} \propto \Phi(R, 0)$$

and this cannot hold everywhere as it results in much too large values of $K_z=\partial\Phi/\partial z$ a short distance above and below the galactic plane at large distances from the centre.

Schmidt attempts to identify some portions of his derived density distribution with observed groups of galactic objects, but there remains a considerable amount of mass to be ascribed to "unknown objects". It is possible that errors in the adopted values of R_0 (8·2 kpc) and the Oort constants of galactic rotation, $A(+19{\cdot}5$ km/sec kpc) and $B(-6{\cdot}9$ km/sec kpc) are connected with this anomaly. We shall here assume that Schmidt's potential function is the best available and we shall endeavour to discover whether Camm's form (9) is suitable for the representation of Schmidt's values.

Kuzmin (**21**) obtains a solution to equation (8) which is identical to Camm's if we transform (ξ, η) to Kuzmin's (ξ_1, ξ_2) by the equations

$$Z_0^2=-2\beta; \quad \xi_1^2-1=\frac{-1}{2\beta}(\xi+\beta); \quad \xi_2^2-1=\frac{-1}{2\beta}(\eta+\beta).$$

His evaluation of this solution as a galactic model is not, however, based on a complete "observational" model such as Schmidt's but is more hypothetical. Further comparison of the present account with Kuzmin's will be made later.

4. *An appropriate value of* β.—For points with $R=0$ the coordinates (ξ, η) are $\xi=-\beta$, $\eta=z^2+\beta$ and for points with $z=0$ they are $\xi=R^2-\beta$, $\eta=\beta$. It is easy to show that if $\Phi=\{F(\xi)-G(\eta)\}/(\xi-\eta)$ the following equation must be satisfied at all points $(R, 0)$:

$$\left(\frac{\partial^2\Phi}{\partial z^2}\right)_{R,0} = \frac{1}{(R^2-2\beta)}\left\{R\left(\frac{\partial\Phi}{\partial R}\right)_{R,0} + \frac{4\beta^2}{R^2-2\beta}\left(\frac{\partial^2\Phi}{\partial z^2}\right)_{0,0} + \frac{4\beta}{R^2-2\beta}[\Phi(0,0)-\Phi(R,0)]\right\}. \quad (13)$$

If we wish to apply this equation for $R=R_0=8{\cdot}2$ kpc we can take the following values for the various differential coefficients from the data of Schmidt's Table 7. Units are 100 km^2/sec^2 or 100 km^2/sec^2 kpc^2.

$$\Phi(0, 0)=-1436 \qquad \Phi(R_0, 0)=-410$$

$$\left(\frac{\partial^2\Phi}{\partial z^2}\right)_{0,0}=12600 \qquad R_0\left(\frac{\partial\Phi}{\partial R}\right)_{R_0,0}=467 \qquad \left(\frac{\partial^2\Phi}{\partial z^2}\right)_{R_0,0}=58.$$

Inserting these values into (13) gives a quadratic for β with one root $\beta=-2{\cdot}25$, and a positive root which is rejected as it would involve difficulties at the point $z=0$, $R^2=2\beta$.

A better value of β near to $-2{\cdot}25$ was then found in the following way. We have

$$\left.\begin{aligned} &\Phi(R,0)=\{F(R^2-\beta)-G(\beta)\}/(R^2-2\beta)\\ \text{and}\quad &\Phi(R_0,z)=\{F(\xi)-G(\eta)\}/(\xi-\eta)\\ \text{where}\quad &\xi+\eta=R_0{}^2+z^2=67{\cdot}24+z^2\\ \text{and}\quad &\xi\eta=\beta(67{\cdot}24-z^2-\beta). \end{aligned}\right\} \qquad (14)$$

Probably the best determined values of Schmidt's Table 7 for Φ are those of $\Phi(R,0)$ for all R from 0 to 16·4 kpc and of $\Phi(R_0,z)$ for z up to 18 kpc. Using equations (14) and adopting trial values of β near to $-2{\cdot}25$, $F(\xi)$ and $G(\eta)$ could be deduced, thus giving

$$\Phi(0,z)=\frac{F(-\beta)-G(z^2+\beta)}{-(z^2+2\beta)}.$$

A comparison of these quantities could then be made with Schmidt's values and, as shown in Fig. 2, the value $\beta=-2{\cdot}5$ gave the best fit. Because the computation of $G(\eta)$ depended upon the difference of two products, the solution

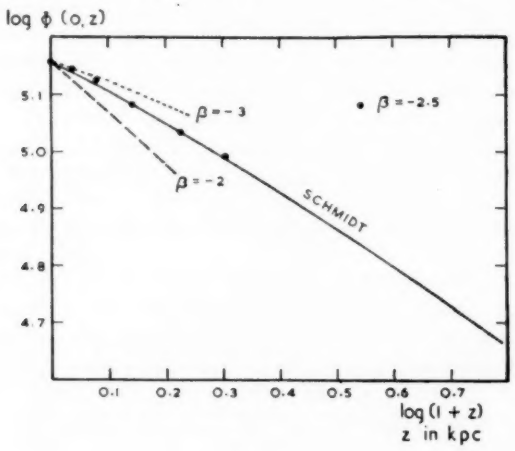

FIG. 2.

became insensitive for points $(0,z)$ beyond $z=1$ kpc and could not be expected in principle to give $\Phi(0,z)$ beyond $z^2=-2\beta$, or $z=2{\cdot}2$ kpc. The point $R=0$, $z=\sqrt{-2\beta}$ is a singularity in the (ξ,η) coordinates.

Slight modifications were needed in Schmidt's values of $\Phi(R_0,z)$ for $z>/4$ kpc to give, with $\beta=-2{\cdot}5$, Schmidt's values of $\Phi(0,z)$ over the range $z=1$ to 2 kpc. A comparison of the two sets of $\Phi(R_0,z)$ is shown in Table I, while Table II gives an abbreviated form of Schmidt's Table 7 and its counterpart, using Camm's form of Φ evaluated as above with $\beta=-2{\cdot}5$.

In Table II it can be seen that the region $0<R<4$ kpc, $z>0$ is not satisfactorily represented. In the region of the singularity at $(0,2{\cdot}2)$ some revision of Schmidt's potential function is required before it will fit Camm's form well, even though

TABLE I

z	$\Phi(R_0, z)$ Schmidt	$\Phi(R_0, z)$ modified
kpc	100 km²/sec²	100 km²/sec²
3	354	354
4	332	332
5	312	310
7	273	269
10	228	227
14	180	171
18	150	136

TABLE II

$-\Phi(R, z)$ *Units:* 100 *km²/sec²*

Schmidt R (kpc) / z (kpc)	0	2·05	4·10	6·15	8·20	10·25	12·30	16·40
0	1436	1012	748	554	410	318	258	188
1	989	851	675	518	396	312	257	188
4	539	520	460	395	332	280	239	183
10	275	270	260	247	228	208	188	156
18	160	160	158	154	150	145	–	–

Camm Function with $\beta = -2{\cdot}5$

R (kpc) / z (kpc)	0	2·05	4·10	6·15	8·20	10·25	12·30	16·40
0	1436	1012	748	554	410	318	258	188
1	980	764	674	525	396	312	254	187
4	539	563	501	414	332	264	231	174
10	275	274	256	239	227	197	179	148
18	160	153	146	138	136	133	126	116

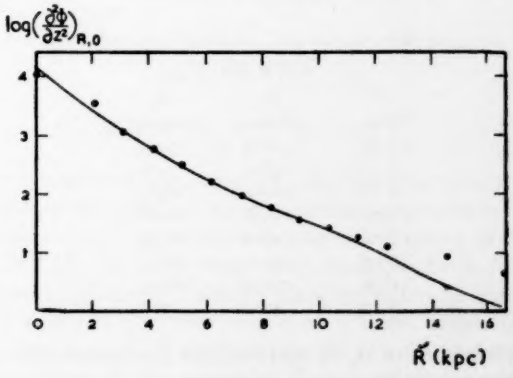

FIG. 3.

elsewhere the fit is good. Kuzmin, by using a particular smooth function for density and potential, avoids difficulty with this singularity.

Another check on the value of β, this time for points $(R, 0)$, may be made by considering how well equation (13) is satisfied. A comparison of the logarithmic values of $\partial^2\Phi/\partial z^2$ found from (13) (filled circles) and from Schmidt's Table direct (continuous curve) is shown in Fig. 3. The agreement is fairly good and neither here nor, for the most part, in Table II are the discordances considered to be large enough to say that the gravitational field of the Galaxy is not such that a system of stars can maintain quite closely a steady state in the way envisaged by Camm.

5. *The axes of the velocity ellipsoid.*—Camm pointed out that in the present analysis knowledge of Φ does not imply the values of the axes of the velocity ellipsoid, which are still left with at least two degrees of freedom. It concerns rather their variation from point to point. Suppose that we have a set of stars in a steady state with axes for the velocity ellipsoid in the Sun's neighbourhood $(R = 8{\cdot}2\,\text{kpc}, z = 0)$

$$\sigma_a = 41\,\text{km/sec}$$
$$\sigma_b = 25\,\text{km/sec}$$
$$\sigma_c = 18\,\text{km/sec}.$$

These figures are taken from Allen (**22**), p. 221. $a = 1/2\sigma_a^2$, etc., and, using equations (5) and $\beta = \frac{1}{2}(\alpha_0 - \alpha_4)/\alpha_3 = -2{\cdot}5$, we obtain

$$\left.\begin{aligned} \alpha_0 &= 2{\cdot}97 \quad 10^{-4} \\ \alpha_2 &= 7{\cdot}48 \quad 10^{-6} \\ \alpha_3 &= 1{\cdot}725 \quad 10^{-5} \\ \alpha_4 &= 3{\cdot}83 \quad 10^{-4} \end{aligned}\right\} \qquad (15)$$

the units for α_0 and α_4 being sec^2/km^2 and for α_2 and α_3, $\text{sec}^2/\text{km}^2\,\text{kpc}^2$. These values (15) are satisfactory in that none are negative and the value of α_1 in (6) may be found by making the maximum value of Θ_0 at points $(R, 0)$ equal to the maximum value of Θ_c. According to (6), for points $(R, 0)$, $\partial\Theta_0/\partial R = 0$ where $R = \sqrt{\alpha_0/\alpha^2} = 6{\cdot}3\,\text{kpc}$, from (15). Θ_c reaches a maximum quite near this value of R and, with $\alpha_1 = 0{\cdot}0212$, Table III gives a comparison of Θ_0 and Θ_c.

TABLE III

R kpc	Θ_c km/sec	Θ_0 km/sec
2·05	178·3	132·1
3·075	198·4	179·4
4·10	211·4	205·3
6·15	224·6	224·6
8·20	216·2	217·0
10·25	192·1	200·3
12·30	171·1	182·3
16·40	142·0	150·3

Assuming that these values of Θ_c are accurate, the consequences of the differences in Table III on a completely steady state picture are shown in Table IV, column 2. Using equation (12) the required values of $(1/\nu)\,\partial\nu/\partial R$ are calculated.

If the system of stars with this velocity function are to give density gradients in accordance, roughly, with the values of Schmidt's Fig. 1 we expect

$$\frac{1}{\nu}\frac{\partial \nu}{dR} \sim -0{\cdot}5$$

over quite a large range of R, since the total density has to decrease by a factor e in about two parsecs; Schmidt discusses the rather high values of $1/\nu\ \partial\nu/\partial R$ that

TABLE IV

R	$\frac{1}{\nu_1}\frac{\partial \nu_1}{\partial R}$	$\frac{1}{\nu_2}\frac{\partial \nu_2}{\partial R}$	$\frac{1}{M}\frac{\partial M}{\partial R}$	$\frac{1}{\nu_3}\frac{\partial \nu_3}{\partial R}$
kpc				
2·05	−4·28	−5·34	−0·32	−4·60
4·10	−0·55	−1·19	−0·32	−0·87
6·15	−0·15	−0·50	−0·32	−0·47
8·20	−0·14	−0·32	−0·30	−0·44
10·25	+0·035	−0·06	−0·18	−0·15
12·30	+0·056	0·00	−0·075	−0·02
16·40	−0·021	−0·04	−0·010	−0·03

occur in his model. We can alter the assumed value of α_1 to remove the positive values in col. 2 of Table IV and obtain those values shown in col. 3. These values are not altogether satisfactory, especially for $R < 5$ kpc, and an attempt was therefore made to see whether a minor modification in the form of Ψ would give any chance of improving the situation.

Suppose we take $\Psi(\Pi, \Theta, Z) = (V^2 - \Pi^2 - \Theta^2 - Z^2)\, e^{-Q}$ where

$$Q = a\Pi^2 + b\Theta^2 - 2b\Theta\Theta_0 + 2g\Pi Z + cZ^2 + \sigma,$$

as before. We may identify V with the escape velocity given by $V^2 = -2\Phi$ so that $\Psi = 0$ when $\Pi^2 + \Theta^2 + Z^2 = V^2$. It proved possible to maintain equations (5) and (6) as the steady-state solutions for a, b, c, g and Θ_0 as the substitution of $Q' = -\ln(V^2 - \Pi^2 - \Theta^2 - Z^2) + Q$ into (4) led to terms being added to the polynomial in Π, Θ and Z (not given here) which were identically zero. An expression for the density was evaluated by making the approximation

$$\iiint_{\Pi^2+\Theta^2+Z^2<V^2} \Psi\, d\Pi\, d\Theta\, dZ = \int_{-\infty}^{\infty}\int\int \Psi\, d\Pi\, d\Theta\, dZ.$$

The expression for ν may be written, for points $(R, 0)$,

$$\nu = e^{(-\sigma + b\Theta_0^2)}\pi M/\sqrt{abc} \tag{16}$$

where

$$M = \sqrt{\pi}[V^2 - \Theta_0^2] - \frac{2\Theta_0}{\sqrt{b}} - \Gamma(\tfrac{3}{2})\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right).$$

In the interpretation of the observed velocities the values of the constants a, b, c, g, and Θ_0 are the same as the corresponding quantities in the true ellipsoidal hypothesis, or very nearly so, since for small values of Π, $\Theta - \Theta_0$, and Z the quantity $(V^2 - \Pi^2 - \Theta^2 - Z^2)$ varies only slowly, even if V is no more than $1{\cdot}4\Theta_0$. The fourth column of Table IV gives the values of $1/M\ \partial M/\partial R$ and the fifth column the values of $1/\nu\ \partial\nu/\partial R$ from (16). There remain the large negative values for $R < 5$ kpc and rather small values for $R > 10$ kpc but it is seen that a relatively small change in the nature of Ψ may affect the satisfaction of this steady-state condition.

For other values of

$$a = \alpha_0 + \alpha_3 z^2$$
$$b = \alpha_0 + \alpha_3 z^2 + \alpha_2 R^2$$
$$c = \alpha_4 + \alpha_3 R^2$$

we may briefly consider the consequences that follow from the relation

$$\alpha_4 - 5\alpha_3 = \alpha_0$$

together with the probability that $\alpha_2/\alpha_0 \sim 0{\cdot}03$ (in the units here employed). The latter condition follows from the necessity for Θ_0 to reach a maximum somewhere near $R = 6$ kpc.

To maintain values of $1/\nu\ \partial\nu/\partial z$ of the order of -4 along the line of points $(0, z)$ it is necessary that there should exist, towards the central regions, a majority of objects with $\alpha_4 \sim 4{\cdot}10^{-5}$, or $\sigma_z \sim 110$ km/sec. We cannot from this put definite values to α_0 and α_3 except as regards their probable order of magnitude. Using the units here employed we expect, numerically, from the form of a, b, and c for large values of R and z, that $0 < \alpha_2 \simeq \alpha_3 \ll \alpha_0$ and hence $\alpha_4 - \alpha_0 = 5\alpha_3 > 0$. We then deduce that $(\alpha_4 + R_0^2\alpha_3)$ should be rather greater than α_0 but only slightly greater than $(\alpha_0 + R_0^2\alpha_2)$ or, in terms of the velocity ellipsoid axes, $\sigma_\Pi > \sigma_\Theta \sim \sigma_Z$. The solution $\alpha_3 = 0$, $\alpha_0 = \alpha_4$ is nevertheless a possible one, giving $\sigma_\Pi = \sigma_Z > \sigma_\Theta$, as in Jeans' solution, though of course the analysis of Camm's solution then does not hold and no value can be assigned to β.

The distribution of the velocities of high-velocity objects is so little known that it does not seem profitable to follow the various possibilities further.

Taking the four values (15) of α_0, α_2, α_3 and α_4 the velocity ellipse in the plane $\theta =$ constant at a point $R = R_0$, $z = 0{\cdot}5$ kpc is

$$3{\cdot}013\Pi^2 - 1{\cdot}414\Pi Z + 15{\cdot}43Z^2 = 10^4.$$

The non-zero value of the coefficient of ΠZ in this equation gives a tilt of the axis $3^\circ\ 15'$ away from the line parallel to the galactic plane. In order to detect such a small tilt it seems that one would need the radial velocities of about 1000 stars at approximately this distance whose proper motions are sufficiently well known for them to be assigned as having a positive or negative Π-value with some certainty. This is rather far beyond the resources of present catalogues.

6. *Conclusions.*—The situation seems to be as follows. Schmidt has built up a model of the Galaxy to fit available data, assumed to be correct. He has constructed it in a semi-empirical way out of relatively few non-homogeneous spheroids of a particular kind of mass-law, superimposing a number of homogeneous spheroids with a small total mass (1 per cent of whole). Camm's functional form for a steady-state galaxy cannot be made to fit analytically the potential function within these spheroids but it is suggested here that the final numerical values for the potential function arrived at by Schmidt do fall roughly into Camm's form, at any rate over the majority of the volume occupied by the Galaxy. Thus it is suggested that the Galaxy has reached a shape in which a steady-state distribution of stellar density and velocity is possible to a fairly high degree of closeness and it has been shown that the ratio of axes of the velocity ellipse for stars in the solar neighbourhood is such as would be expected for such a distribution. Kuzmin (**23**) arrives at a similar conclusion by a slightly different process. His idealized models are effectively such as to allow a steady-state distribution but it is not clear whether his superposition of spheroids does in fact allow it.

The difficulty of satisfying Poisson's equation, discussed analytically by Camm, has not, so far, been mentioned. It is impossible for a group of stars in a steady state with a uniform mass-distribution to produce a gravitational field in which a steady-state motion of the type discussed here is possible. The only explanation, as applied to the Galaxy, is to assume that the superposition of different systems within the Galaxy with different mass frequency-distributions nevertheless does allow a steady state. Given what is known of the potential function of the Galaxy, non-equality of the radial and z axes of the velocity ellipsoid near the Sun is not inconsistent with the steady-state hypothesis. Other observed features, of course, are, but they are more concerned with details than with the overall gravitational field. It is important in investigating non-steady features that we should be clear as to what features do in fact represent departures from the steady-state hypothesis.

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Radcliffe Observatory,
Pretoria:
1958 *December.*

References

(1) J. H. Oort, *Scientific American*, **195**, 100, 1956.
(2) O. J. Eggen, *M.N.*, **118**, 65 and 154, 1958.
(3) K. Schwarzschild, *Gött. Nach*, p.614, 1907 and p. 191, 1908.
(4) J. H. Jeans, *M.N.*, **76**, 70, 1915.
(5) W. M. Smart, *Stellar Dynamics*, p. 338, Camb. Univ. Press, 1938.
(6) J. H. Oort, *B.A.N.*, **4**, 269, 1928; No. 159.
(7) M. Schmidt, *B.A.N.*, **13**, 15, 1956; No. 468.
(8) R. v. d. R. Woolley, *M.N.*, **117**, 198, 1957.
(9) A. S. Eddington, *M.N.*, **76**, 37, 1915.
(10) G. L. Clark, *M.N.*, **97**, 182, 1937.
(11) S. Chandrasekhar, *M.N.*, **98**, 710, 1938.
(12) G. L. Camm, *M.N.*, **101**, 195, 1941.
(13) P. P. Parenago, *Astr. Journ. U.S.S.R.*, **29**, 245, 1952.
(14) A. S. Eddington, Halley Lecture, 1930.
(15) S. Chandrasekhar, *Ap. J.*, **90**, 43, 1939.
(16) B. Lindblad, *M.N.*, **87**, 553, 1927, *inter alia.*
(17) G. Stromberg, *Ap. J.*, **59**, 228, 1924.
(18) J. H. Oort, *B.A.N.*, **6**, 249, 1932; No. 238.
(19) K. K. Kwee, C. A. Muller, and G. Westerhout, *B.A.N.*, **12**, 211, 1954; No. 458.
(20) S. Chandrasekhar, *Ap. J.*, **90**, 53, 1939.
(21) G. Kuzmin, *Astr. Journ. U.S.S.R.*, **33**, 27, 1956. (Tartu Teated No. 2.)
(22) C. W. Allen, *Astrophysical Quantities*, Athlone Press, 1955.
(23) G. Kuzmin, Tartu Teated No 3.

THE RADIO EMISSION FROM THE DIRECTION OF THE "SUPERGALAXY"

J. R. Shakeshaft and J. E. Baldwin

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Summary

Observations have been made with the Cambridge radio telescope at a wave-length of 1·9 m from which radio isophotes of the region of sky 10^h–14^h have been derived in order to investigate the features which have been ascribed to radiation from the "local supergalaxy". The distribution of emission is not closely correlated with counts of galaxies. The observed intensities are several times greater than those to be expected from normal galaxies and counts of radio sources show that this factor cannot be explained by a small proportion of the galaxies having abnormally high radio luminosities. The radio brightness of one of the features is very large compared with the total background of extra-galactic emission. It is suggested that the radio features are not in fact associated with the "supergalaxy".

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1. *Introduction.*—Surveys of the background radiation by Hanbury Brown and Hazard (1953) at 159 Mc/s and Kraus and Ko (1953) at 242 Mc/s have revealed that there is a bright band of radio emission running roughly perpendicular to the galactic plane at about 12^h R.A. These authors have suggested that this band represents the integrated emission from the concentration of bright galaxies lying along a great circle crossing the galactic plane at longitudes $l = 105°$ and $285°$.

De Vaucouleurs (1953, 1956, 1958) has presented evidence to show that these galaxies and our own are members of a flattened system or "cluster of clusters" with its centre near the Virgo cluster. This he terms the Local Supergalaxy, after Shapley. In his later paper the radio data are summarized as additional support for this picture, but it is the purpose of the present paper to show that there are certain difficulties in such an interpretation of the radio feature.

2. *The shape and position of the radio feature in relation to the counts of galaxies.*—One of the four sections of the Cambridge four-aerial interferometer (Ryle and Hewish 1955) has recently been used for a survey of the background radiation at a frequency of 159 Mc/s. The beam-width between half power points is $1°{\cdot}2$ in right ascension and $7°$ in declination. Contours of brightness temperature derived from these measurements are shown in Fig. 1, plotted together with the positions of galaxies from the Shapley–Ames catalogue (1932). This catalogue contains the positions of galaxies brighter than about $m_{pg} = 13$ and is believed to be substantially complete to that level.

In the map of Fig. 1 the areas of greatest brightest temperature are shown hatched. The contours are noticeably affected by the presence of two bright radio sources. One of these, at $\alpha\, 12^h\, 28^m$, $\delta + 12°$, is Virgo A and the other is Cassiopeia A which, observed at lower culmination in a 1 per cent sidelobe of

the declination reception pattern of the aerial, causes the long ridge extending from $\alpha\ 12^h\ 10^m$, $\delta + 50°$ to $\alpha\ 11^h\ 50^m$, $\delta + 08°$.

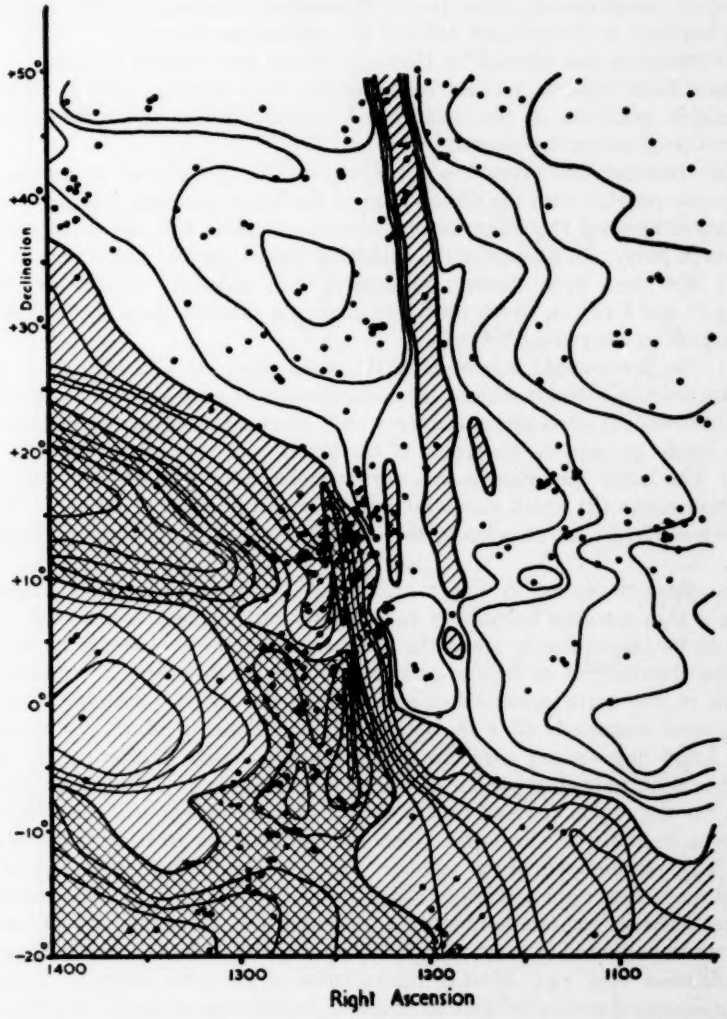

FIG. 1.—*Contours of aerial temperature at 159 Mc/s plotted together with the positions of galaxies from the Shapley–Ames catalogue. The contour interval is approximately 8°K.*

Apart from these sources, the main features to be seen on the map are:

(i) A general increase in brightness from $\alpha\ 10^h\ 30^m$–$14^h\ 00^m$ associated with the extended component of the galactic radio emission.

(ii) An intense ridge between $\delta + 08°$ and $+27°$ extending from $\alpha\ 14^h\ 00^m$ to $12^h\ 30^m$. This is part of a much larger feature, detected in many surveys, which runs from the galactic plane near $l = 0°$ towards the north galactic pole.

(iii) A bright region between $\delta -15^\circ$ and $+05^\circ$ and $\alpha\ 12^h15^m$ to 13^h00^m. It is this feature that Kraus and Ko (1953) believe to be associated with emission from the Supergalaxy.

(iv) A comparatively faint ridge of emission running roughly north and south between $\alpha\ 11^h00^m$ and 12^h30^m at declinations north of $\delta +15^\circ$. Part of this feature is that studied by Hanbury Brown and Hazard (1953).

Since faint features are not easily visible on a contour map if there are appreciable gradients in the background, sections across the map at several different declinations are plotted in Fig. 2.

The enhanced radio emission in the regions (iii) and (iv) above can be seen to coincide roughly with the distribution of the bright galaxies, but the detailed correlation between the intensity of the radio emission and the density of the galaxies is poor. In particular the following points may be noted:

(*a*) The area most densely populated with galaxies, centred on about $\alpha\ 12^h30^m$ and $\delta\ 12^\circ$, in which the Virgo cluster is situated, does not correspond with a peak in the radio isophotes.

(*b*) The pronounced maximum of the radio isophotes at $\alpha\ 12^h40^m$, $\delta -08^\circ$ does not seem to coincide with a high concentration of galaxies. De Vaucouleurs (1956) shows that obscuration can have little effect on the apparent distribution of the bright galaxies in this region of the sky.

(*c*) The latter maximum seems, furthermore, to be connected to the ridge of high intensity (ii) which shows no correlation with counts of galaxies. There is strong evidence for associating the latter feature with the Galaxy (Baldwin 1955).

The galaxies plotted in Fig. 1 are mainly those of the Local Supergalaxy having a high intrinsic luminosity and an apparent magnitude less than +13. It would be interesting to see if the radio isophotes were in better agreement with the distribution of fainter galaxies. There is an excess in numbers of galaxies in the north galactic hemisphere over the south, persisting down to an apparent magnitude of +18, which de Vaucouleurs attributes to members of the Local Supergalaxy with low intrinsic luminosities. Unfortunately, little work has been published on the distribution of galaxies in different magnitude ranges for the region covered in Fig. 1 and such information is available only for a zone from $\delta +41^\circ$ to $+46^\circ$. Data for this zone are given by Shapley and Jones (1940) in the form of the number of galaxies in nine square degrees of sky centred on various right ascensions along declination $+43^\circ 30'$. These numbers for three different magnitude ranges are plotted in Fig. 3 together with the aerial temperature measured at a wave-length of 1·9 m with a reception pattern centred on declination $+42^\circ 14'$. Most of the variation of the radio intensity is caused by the main component of the galactic emission, but the feature under consideration can be seen at $\alpha\ 12^h$. It will be noticed that the belt of excess radiation coincides with one of the smaller increases in number density of the fainter galaxies. The emission associated with the considerably larger peaks in the galactic counts at 13^h and 16^h is certainly not greater than 20 per cent of that at 12^h.

Since the aerial beam has a width to half-power points of 7° in declination it extends beyond the limits of the areas of nine square degrees used for the galaxy counts. The counts of galaxies in areas of 25 square degrees given by Shapley and Jones were therefore compared with the radio observations. The results were substantially the same as those shown in Fig. 3.

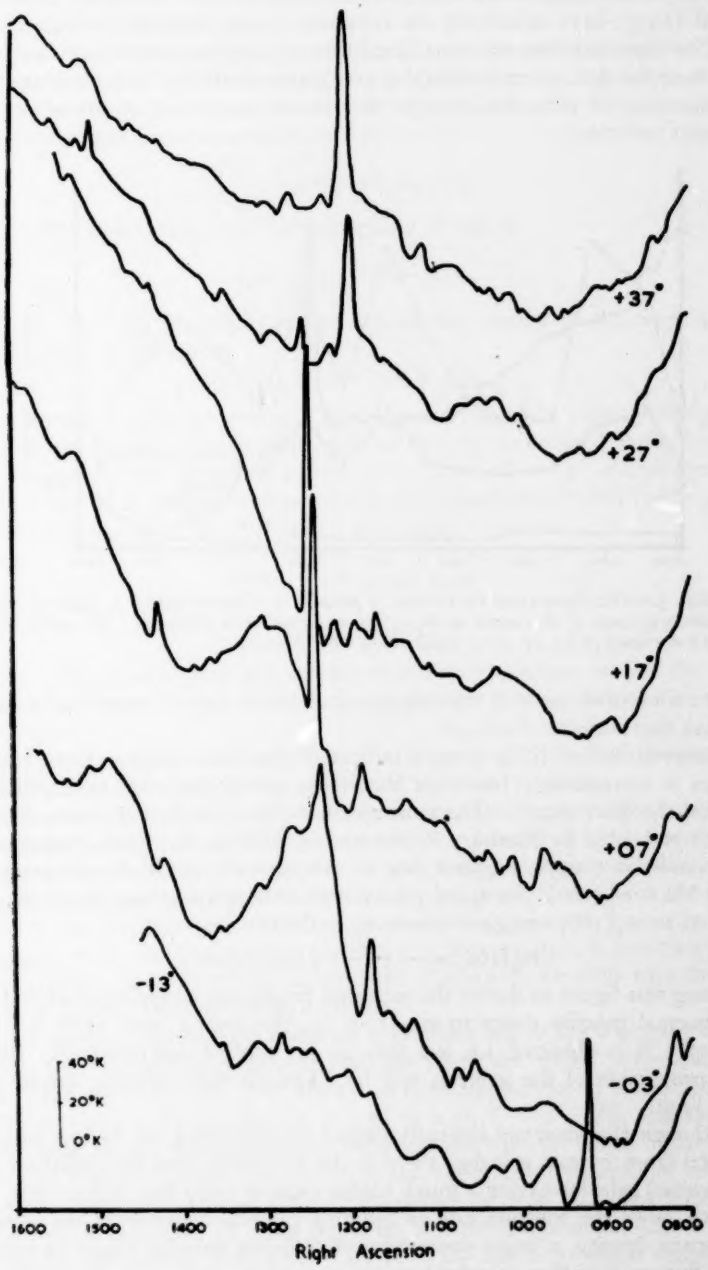

FIG. 2.—*Variation of the aerial temperature with right ascension for a number of different declinations.*

3. *The radio emission expected from the Supergalaxy.*—Hanbury Brown and Hazard (1953) have calculated the intensity of the integrated emission which would be expected from the normal galaxies in a region near $\alpha = 12^h$, $\delta = +43°$. They used the data given by Shapley and Jones, assuming that all galaxies have the same ratio of radio flux to light flux as that found for spirals not emitting enhanced radiation.

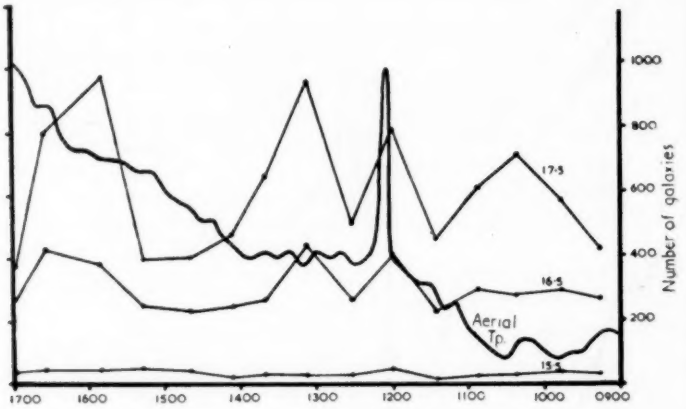

FIG. 3.—*This figure shows the numbers of galaxies in 9 square degrees to different limiting magnitudes in areas of sky centred on* $\delta + 43° 30'$ *at various right ascensions. Also plotted is the aerial temperature at* $\delta + 42° 14'$ *as a function of right ascension.*

The computed value of the integrated emission was 1·7 magnitudes (4·8 ×) less than that actually observed.

Observations by Mills (1955) indicated that the emission from elliptical galaxies is considerably less than that from spirals and this result therefore increases the discrepancy. The most recent work on the flux densities of normal galaxies presented by Hanbury Brown and by Mills at the Paris symposium on radio astronomy, 1958, suggests that an average value for the radio magnitude at 160 Mc/s, $m_R(160)$ of a spiral galaxy with photographic magnitude m_{pg} may be taken as $m_R(160) - m_{pg} \geqslant 0$ where m_R is defined by

$$m_R(160) = -53{\cdot}4 - 2{\cdot}5 \log_{10} S(160).$$

Using this figure to derive the expected brightness temperature at 160 Mc/s from normal galaxies down to m_{pg} 17·5 in the region near $\alpha 12^h$, $\delta + 43°$ a value of 5 °K is obtained, i.e. less than 20 per cent of that observed. Because some proportion of the galaxies will be ellipticals this estimate should be an upper limit.

Although the observed intensity cannot be accounted for by the integrated emission from normal galaxies, there is the possibility that the radiation arises in abnormal galaxies having a much higher ratio of radio flux to light flux. For instance, since the collision rate of colliding galaxies depends on the square of their space density, a larger proportion of colliding galaxies might be expected in the Supergalaxy than in neighbouring areas of the sky.

In this case, if the observed excess brightness temperature in the belt is to be explained as the integrated emission from such sources, then a consideration

of the actual numbers of radio sources observed in the region enables a lower limit to be placed on their space density.

Suppose that the maximum excess brightness temperature of 80 °K is the integrated effect of discrete sources each having an equal luminosity P w. ster^{-1} (c/s)$^{-1}$ with a space density of ρ Mpc^{-3}, then the flux density per steradian from distances between R_0 and R_2 is

$$\int_{R_1}^{R_2} \rho \frac{r^2 P}{r^2} dr = \rho P(R_2 - R_1)$$

and the brightness temperature corresponding to this is

$$T_b = \frac{c^2}{2k\nu^2} \rho P(R_2 - R_1).$$

Taking $(R_2 - R_1)$ as $\leqslant 15$ Mpc, the approximate extent of the Supergalaxy according to de Vaucouleurs

$$P\rho \geqslant 1{\cdot}4 \times 10^{-45} \text{ MKS units.}$$

The survey of radio sources at a frequency of 159 Mc/s recently completed at Cambridge shows five sources having a flux density $> 15 \times 10^{-26}$ w.m.$^{-2}$ (c/s)$^{-1}$ in the area of sky $12^h\,00^m < \alpha < 13^h\,30^m$, $-10° < \delta < +4°$ (approximately $\frac{1}{12}$ steradian). If all of these were sources in the Supergalaxy* then $\frac{1}{3}\rho R^3 = 5 \times 12$ where R is related to P by $P/R^2 = 15 \times 10^{-26}$ w.m.$^{-2}$ (c/s)$^{-1}$.

Thus

$$\rho P^{3/2} \simeq 10^{-35} \text{ MKS units}$$

whence

$$\rho \geqslant 700 \text{ Mpc}^{-3},$$

which is much greater than the density of ordinary galaxies, so that the radio isophotes cannot be explained in terms of galaxies having abnormally strong radio emission.

This calculation ignores the effect of a dispersion in absolute luminosities which would lead to a still greater density of abnormal sources.

Shklovsky (1954), using different arguments, also came to the conclusion that emission from individual sources could not account for the feature; he suggested that the emission was due to relativistic electrons accelerated by inter-galactic magnetic fields of the order of 10^{-7} gauss throughout the whole volume of the Supergalaxy. This is not a hypothesis that can be easily tested but, in common with all the other theories that the radiation is associated with the Supergalaxy, it is, as will be shown later, difficult to reconcile with the total amount of the extra-galactic radio emission.

Some further experimental evidence is available which has a bearing on this matter. De Vaucouleurs (1956) shows that there are other "supergalaxies" not far from the local one. The nearest of these appears as an elliptical swarm of galaxies running from $\alpha\ 3^h\,30^m$, $\delta\ -7°$ to $\alpha\ 5^h$, $\delta\ -60°$. He suggests that it is seen edge-on and that it is perhaps rather smaller than the Local Supergalaxy. Bolton and Westfold (1950) have surveyed this region at a frequency of 100 Mc/s with a 17° beamwidth and an estimated accuracy of ±20 °K. The swarm lies right across a minimum of their isophotes but no irregularities can be seen. The angular dimensions of the swarm are $10° \times 60°$ and so an aerial temperature of about 100°–200° K might be expected.

* Since the Supergalaxy is not apparent on source maps only 1 or 2 of the 5 could be associated with it.

The central part of the swarm is, unfortunately, not visible from Cambridge, but some of the scans at a frequency of 159 Mc/s cross it at a place where the concentration of bright galaxies is still marked. Fig. 4 shows the variation of aerial temperature with right ascension at $\delta -28°$. The position of the supergalaxy in question is also indicated and it can be seen that the upper limit to the aerial temperature from it is about 5 °K.

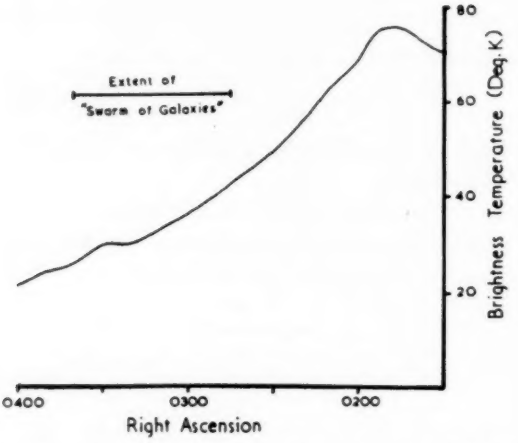

FIG. 4.—*The variation of temperature at 159 Mc/s and* $\delta -28°$ *with right ascension. Also marked is the position of a swarm of galaxies noted by de Vaucouleurs.*

An examination may also be made of dense clusters of galaxies which, on Shklovsky's hypothesis, should be very intense sources. The most readily detectable should be those in Perseus and Coma. Data on the numbers and types, e.g. spirals, ellipticals, etc., of galaxies in these clusters are meagre and it is not possible to calculate accurately the flux expected if there are no sources with enhanced emission. For a very rough estimate let us assume that there are 1000 galaxies in each cluster, with a mean m_{pg} of 16. Assuming also that the relationship $m_R(160) - m_{pg} = 0$ may be applied, then the expected flux density at 160 Mc/s is 16×10^{-26} w.m.$^{-2}$ (c/s)$^{-1}$.

If this value is compared with the observed flux density at 81·5 Mc/s for the Perseus cluster of 45×10^{-26} w.m.$^{-2}$ (c/s)$^{-1}$ after the contribution from NGC 1275 has been subtracted (Baldwin and Elsmore 1954), and of $10{\cdot}7 \times 10^{-26}$ w.m.$^{-2}$ (c/s)$^{-1}$ at 400 Mc/s for the Coma cluster (Seeger, Westerhout and Conway 1957), the discrepancies do not seem to justify the belief that there is a large excess of radiation from these clusters.

4. *The integrated extra-galactic component.*—In the brightest part of the radio feature the excess radiation (about 80 °K at 160 Mc/s) amounts to more than 20 per cent of the total intensity in that direction. Most of the background emission is, however, galactic in origin. An upper limit on the amount of the extra-galactic component has been set by Baldwin (1955) from an analysis of the angular distribution of the galactic background. This limit is 500 °K at 81·5 Mc/s. Although the spectrum of the extra-galactic component is at present unknown, we can use the 81·5 Mc/s observations to show that the excess radiation from the

direction of the Supergalaxy is of the same order of magnitude as the total extra-galactic radiation.

If the radiation originates in the Local Supergalaxy, then the first 15 Mpc of space measured in a line of sight from the Earth in the direction of the Supergalaxy must be intrinsically different from, say, the next 1000 Mpc. In view of the large scale uniformity found among the more distant galaxies, such a state of affairs seems unlikely.

5. *Conclusions.*—The evidence presented in this paper does not support the physical association of the radio feature with the Local Supergalaxy and it seems possible that it originates within the galaxy. It lies close to the much more intense belt of radiation which is very probably galactic. The radiation from this latter feature has been discussed by Mrs H. Tunmer (1958).

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Mullard Radio Astronomy Observatory,
Cavendish Laboratory,
Cambridge:
1958 *December* 23.

References

Baldwin, J. E. and Elsmore, B., 1954, *Nature*, **173**, 818.
Baldwin, J. E., 1955, *M.N.*, **115**, 690.
Bolton, J. G. and Westfold, K. C., 1950, *Aust. J. Sci. Res.*, A, **3**, 19.
Brown, R. Hanbury, and Hazard, C., 1953, *Nature*, **172**, 997.
Kraus, J. D., and Ko, H. C., 1953, *Nature*, **172**, 538.
Mills, B. Y., 1955, *Aust. J. Phys.*, **8**, 368.
Ryle, M. and Hewish, A., 1955, *Mem. R.A.S.*, **67**, 97.
Seeger, C. L., Westerhout, G. and Conway, R. G., 1957, *Ap. J.*, **126**, 585.
Shapley, H. and Ames, A., 1932, *Harvard Annals*, **88**, No. 2.
Shapley, H. and Jones, R. B., 1940, *Proc. Nat. Acad. Sci.*, **26**, 554.
Shklovsky, I. S., 1954, *A.J.U.S.S.R.*, **31**, 533.
Tunmer, H., 1958, *Phil. Mag. Ser.* 8, **3**, 370.
Vaucouleurs, G. de, 1953, *A.J.*, **58**, 30.
Vaucouleurs, G. de, 1956, *Vistas in Astronomy, Vol.* II, Pergamon Press, London.
Vaucouleurs, G. de, 1958, *Nature*, **182**, 1478.

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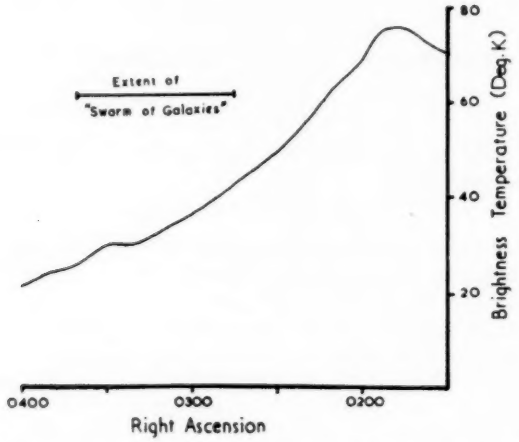

FIG. 4.—*The variation of temperature at 159 Mc/s and* $\delta -28°$ *with right ascension. Also marked is the position of a swarm of galaxies noted by de Vaucouleurs.*

An examination may also be made of dense clusters of galaxies which, on Shklovsky's hypothesis, should be very intense sources. The most readily detectable should be those in Perseus and Coma. Data on the numbers and types, e.g. spirals, ellipticals, etc., of galaxies in these clusters are meagre and it is not possible to calculate accurately the flux expected if there are no sources with enhanced emission. For a very rough estimate let us assume that there are 1000 galaxies in each cluster, with a mean m_{pg} of 16. Assuming also that the relationship $m_R(160) - m_{pg} = 0$ may be applied, then the expected flux density at 160 Mc/s is 16×10^{-26} w.m.$^{-2}$(c/s)$^{-1}$.

If this value is compared with the observed flux density at 81·5 Mc/s for the Perseus cluster of 45×10^{-26} w.m.$^{-2}$(c/s)$^{-1}$ after the contribution from NGC 1275 has been subtracted (Baldwin and Elsmore 1954), and of $10{\cdot}7 \times 10^{-26}$ w.m.$^{-2}$(c/s)$^{-1}$ at 400 Mc/s for the Coma cluster (Seeger, Westerhout and Conway 1957), the discrepancies do not seem to justify the belief that there is a large excess of radiation from these clusters.

4. *The integrated extra-galactic component.*—In the brightest part of the radio feature the excess radiation (about 80 °K at 160 Mc/s) amounts to more than 20 per cent of the total intensity in that direction. Most of the background emission is, however, galactic in origin. An upper limit on the amount of the extra-galactic component has been set by Baldwin (1955) from an analysis of the angular distribution of the galactic background. This limit is 500 °K at 81·5 Mc/s. Although the spectrum of the extra-galactic component is at present unknown, we can use the 81·5 Mc/s observations to show that the excess radiation from the

direction of the Supergalaxy is of the same order of magnitude as the total extra-galactic radiation.

If the radiation originates in the Local Supergalaxy, then the first 15 Mpc of space measured in a line of sight from the Earth in the direction of the Supergalaxy must be intrinsically different from, say, the next 1000 Mpc. In view of the large scale uniformity found among the more distant galaxies, such a state of affairs seems unlikely.

5. *Conclusions.*—The evidence presented in this paper does not support the physical association of the radio feature with the Local Supergalaxy and it seems possible that it originates within the galaxy. It lies close to the much more intense belt of radiation which is very probably galactic. The radiation from this latter feature has been discussed by Mrs H. Tunmer (1958).

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Mullard Radio Astronomy Observatory,
Cavendish Laboratory,
Cambridge:
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References

Baldwin, J. E. and Elsmore, B., 1954, *Nature*, **173**, 818.
Baldwin, J. E., 1955, *M.N.*, **115**, 690.
Bolton, J. G. and Westfold, K. C., 1950, *Aust. J. Sci. Res.*, A, **3**, 19.
Brown, R. Hanbury, and Hazard, C., 1953, *Nature*, **172**, 997.
Kraus, J. D., and Ko, H. C., 1953, *Nature*, **172**, 538.
Mills, B. Y., 1955, *Aust. J. Phys.*, **8**, 368.
Ryle, M. and Hewish, A., 1955, *Mem. R.A.S.*, **67**, 97.
Seeger, C. L., Westerhout, G. and Conway, R. G., 1957, *Ap. J.*, **126**, 585.
Shapley, H. and Ames, A., 1932, *Harvard Annals*, **88**, No. 2.
Shapley, H. and Jones, R. B., 1940, *Proc. Nat. Acad. Sci.*, **26**, 554.
Shklovsky, I. S., 1954, *A.J.U.S.S.R.*, **31**, 533.
Tunmer, H., 1958, *Phil. Mag. Ser.* 8, **3**, 370.
Vaucouleurs, G. de, 1953, *A.J.*, **58**, 30.
Vaucouleurs, G. de, 1956, *Vistas in Astronomy, Vol.* II, Pergamon Press, London.
Vaucouleurs, G. de, 1958, *Nature*, **182**, 1478.

THE RED SHIFT—MAGNITUDE RELATION IN OBSERVATIONAL COSMOLOGY

W. Davidson

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Summary

A reappraisal is made of earlier theoretical work on the red shift—magnitude relation. New formulae are derived which are particularly directed to the practical solution of the cosmological problem and the detection of evolutionary effects.

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1. *Introduction—the cosmological metric.*—Following earlier writers we adopt as the theoretical basis for cosmological models a space-time description of cosmic events characterized by a world metric given by

$$dS^2 = c^2\,dt^2 - R^2(t)\left\{\frac{dr^2 + r^2\,d\theta^2 + r^2\sin^2\theta\,d\phi^2}{(1 + kr^2/4)^2}\right\}. \qquad (1.1)$$

This metric was derived by purely geometrical and kinematic considerations by Robertson (**1**), and also by Walker (**2**), based on the strict application of the cosmological principle.

The cosmological principle is founded essentially on an assumption derived from observations—that as regards the large scale features of the universe there is, at all epochs, spherical symmetry spatially about every 'fundamental' observer with respect to whom the average motion of matter in his (sufficiently large) neighbourhood is zero. In applying the metric (1.1) to the actual universe it has to be supposed that local irregularities have been smoothed out and ideal spatial isotropy exists about every observer co-moving with the matter in his neighbourhood.

The variable t measures cosmic time as it would be registered on a clock carried by such an observer, and $R(t)$ is a disposable function of t whose geometrical interpretation is such that $k/R^2(t)$ is the uniform curvature of space (t = constant) at epoch t. The parameter k is a constant taking the values 1, 0 or -1 according as this curvature is positive, zero, or negative respectively. Fundamental observers co-moving with the matter of the universe are characterized by having constant spatial coordinates r, θ, ϕ.

In observational cosmology the observable quantities employed by astronomers are related as far as possible to the theoretical parameters $R(t)$ and k occurring in the cosmological metric (1.1). Relations are then developed between the observable quantities corresponding to different values of $R(t)$ and k. With these the observed relations may be compared and so the means are provided to determine those values of $R(t)$ and k which best fit the large scale features of the actual universe.

Since exact relations between the observables do not lend themselves to practical tests we shall adopt for the red shift—magnitude relation the method of

expansion in series which has been followed by previous writers, notably McVittie (3), (12), Heckmann (4) and Robertson (5). For this purpose it is convenient to replace coordinate r in (1.1) by means of the substitution

$$\rho = R_0 r, \tag{1.2}$$

where $R_0 \equiv R(t_0)$. We take t_0 to be the present epoch of observation and regard it as fixed once and for all so that R_0 is a constant. The coordinate ρ will have the dimension of distance. The metric (1.1) can now be written

$$dS^2 = c^2\,dt^2 - \frac{R^2(t)}{R_0^2}\left\{\frac{d\rho^2 + \rho^2\,d\theta^2 + \rho^2 \sin^2\theta\,d\phi^2}{(1+\rho^2/a^2)^2}\right\} \tag{1.3}$$

where $a^2 = 4R_0^2/k$.

Putting $t_0 - t \equiv \tau$ we write

$$\begin{aligned} R(t) &= R(t_0 - \tau) \\ &= R_0(1 - \alpha_1\tau + \tfrac{1}{2}\alpha_2\tau^2 - \dots) \end{aligned} \tag{1.4}$$

where $\alpha_1 = \dot{R}_0/R_0$, $\alpha_2 = \ddot{R}_0/R_0$, and so on, a dot indicating differentiation with respect to t.

A light ray in the space-time metric (1.3) is characterized by the equation $dS^2 = 0$, and a photon has the local velocity c relative to a fundamental observer of constant spatial coordinates ρ, θ, ϕ. Accordingly, the light ray which at epoch t is emitted from a source at (ρ, θ, ϕ), and at epoch t_0 reaches an observer at $\rho = 0$, will by symmetry move along the radial line $\theta = \text{constant}$, $\phi = \text{constant}$, such that

$$cR_0 \int_t^{t_0} \frac{dt}{R(t)} = \int_0^{\rho} \frac{d\rho}{1+\rho^2/a^2}. \tag{1.5}$$

By expansion of the integrands and integration term by term we find

$$c\tau\{1 + \tfrac{1}{2}\alpha_1\tau + \tfrac{1}{6}(2\alpha_1^2 - \alpha_2)\tau^2 + \dots\} = \rho - \frac{\rho^3}{3a^2} + \dots,$$

so that on solving for ρ we obtain the useful relation

$$\rho = c\tau\left[1 + \tfrac{1}{2}\alpha_1\tau + \left\{\tfrac{1}{6}(2\alpha_1^2 - \alpha_2) + \frac{c^2}{3a^2}\right\}\tau^2 + \dots\right]. \tag{1.6}$$

2. *Red shift or Doppler effect.*—It is now well known that in those models of metric (1.3) for which the factor $R(t)$ increases monotonically with epoch a shift in wave-length towards the red end of the spectrum will be observed in the light from distant sources. This was first shown by Lemaitre (6). In fact, if at epoch t a source of constant ρ, θ, ϕ emits light of frequency ν and wave-length λ according to a fundamental observer at the source, then the frequency ν_0 and wave-length λ_0 of the light at epoch t_0, when it is registered by another fundamental observer, will be given by

$$\frac{\lambda_0}{\lambda} = \frac{\nu}{\nu_0} = \frac{R(t_0)}{R(t)}. \tag{2.1}$$

If the characteristic emissions of an atom are assumed to be of frequency independent of epoch, then the observer receiving the light will, if $R(t_0) > R(t)$, declare a shift of the spectrum towards the longer wave-lengths by a constant fraction

$$\delta = \frac{R(t_0)}{R(t)} - 1, \tag{2.2}$$

independent of wave-length.

According to the metric (1.3) a progressive increase with epoch of the factor $R(t)$ means a progressive increase in the distance between any two neighbouring sources of constant ρ, θ, ϕ, if we interpret the metric locally in terms of special relativity. The red shifts actually observed by astronomers are now generally interpreted in terms of a Doppler effect arising from recession of the sources.

By expansion of the right-hand side of (2.2) in powers of τ we can write, using (1.4)

$$\delta = \alpha_1\tau + (\alpha_1^2 - \tfrac{1}{2}\alpha_2)\tau^2 + (\alpha_1^3 - \alpha_1\alpha_2 + \tfrac{1}{6}\alpha_3)\tau^3 + \ldots$$

so that, inverting this relationship, we derive

$$\tau = \left(\frac{1}{\alpha_1}\right)\delta + \left(\frac{\tfrac{1}{2}\alpha_2 - \alpha_1^2}{\alpha_1^3}\right)\delta^2 + \left(\frac{\alpha_1^4 - \alpha_1^2\alpha_2 + \tfrac{1}{2}\alpha_2^2 - \tfrac{1}{6}\alpha_1\alpha_3}{\alpha_1^5}\right)\delta^3 + \ldots. \qquad (2.3)$$

This result will be employed later.

3. *Astronomical distance, bolometric and heterochromatic magnitudes.*—The distance normally assigned to a luminous object by astronomers is determined by its apparent bolometric luminosity, assuming an inverse square law of diminution of intensity and given the absolute power of the source. Thus, if $E^*(t)$ is the energy radiated at epoch t from the source in unit time over all frequencies, while l^*_{bol} is the received energy flux per unit area outside the Earth's atmosphere (before suffering heterochromatic absorption), then the luminosity distance D is given by

$$l^*_{\mathrm{bol}} = \frac{E^*(t)}{4\pi D^2}. \qquad (3.1)$$

If m_{bol} is the bolometric apparent magnitude of the source, and M_{bol} is its absolute magnitude corresponding to a luminosity L^*_{bol} at a distance of 10 parsecs, the absolute magnitude of a source being by convention the apparent magnitude when at 10 parsecs, we can write

$$m_{\mathrm{bol}} - M_{\mathrm{bol}} = 2{\cdot}5 \log_{10}(L^*_{\mathrm{bol}}/l^*_{\mathrm{bol}}). \qquad (3.2)$$

Hence, if D is in parsecs,

$$\log_{10} D = 0{\cdot}2(m_{\mathrm{bol}} - M_{\mathrm{bol}}) + 1. \qquad (3.3)$$

Since we are unable to measure directly the absolute magnitude M_{bol} of a very distant source at the time of emission of its light, a relation such as (3.3) is not practical for the cosmological problem. However we can observe *local* sources of the same type and obtain at 10 parsecs

$$L^*_{0(\mathrm{bol})} = \frac{E^*(t_0)}{4\pi 10^2}. \qquad (3.4)$$

Therefore, if $M_{0(\mathrm{bol})}$ is the corresponding absolute magnitude of such a local source, we can write instead of (3.2)

$$m_{\mathrm{bol}} - M_{0(\mathrm{bol})} = 2{\cdot}5 \log_{10}(L^*_{0(\mathrm{bol})}/l^*_{\mathrm{bol}}), \qquad (3.5)$$

where m_{bol} and l^*_{bol} refer to the distant source. It follows from (3.1), (3.4) and (3.5) that

$$\log_{10} D = 0{\cdot}2\left\{m_{\mathrm{bol}} - M_{0(\mathrm{bol})} + 2{\cdot}5 \log_{10}\left(\frac{E^*(t)}{E^*(t_0)}\right)\right\} + 1. \qquad (3.6)$$

The introduction of a theoretical luminosity distance in the cosmological problem arising from the assumption of an inverse square law of diminution of luminosity is due to Whittaker (**7**). The formula for D appropriate to the general metric (1.1) was derived by McCrea (**8**) who obtained, using the notation of our metric (1.3),

$$D = \frac{R_0 \rho}{R(t)(1+\rho^2/a^2)}. \tag{3.7}$$

Thus D represents the luminosity distance from the origin $\rho = 0$ of a source at (ρ, θ, ϕ) whose light was emitted at epoch t and is seen at the origin at epoch t_0.

At the time when McCrea obtained this result the depths of space so far explored were not sufficient to justify specific concern with the possibility of an evolutionary variation of the emission from a typical galaxy. The detection of such an effect has now however become of great importance to distinguish an evolutionary model (see e.g. Tolman (**9**)) from the steady state models of Bondi and Gold (**10**) and of Hoyle (**11**). It is this problem that is our special concern in this paper as it seems to the writer that recent treatments are not altogether satisfactory since they leave obscurities regarding the practical determination of evolutionary effects.

A formula connecting red shift with bolometric magnitude has been given by Robertson (**5**). This relation is essentially a series expansion of our equation (3.6) with D given by (3.7) expanded in powers of δ. It is this formula that is adopted by Sandage as the theoretical basis for his analysis of the red shift data obtained by Humason, Mayall and Sandage (**13**).

The magnitudes directly measured by the astronomers are not of course bolometric, corresponding to total energy received outside the Earth's atmosphere (we neglect galactic obscuration as a latitude effect irrelevant to the present issue), but 'heterochromatic' arising from the selective effects of the atmosphere and sensitivity of photographic plates and other detective devices. Accordingly, since it has been the custom in the literature to express the theoretical relations in terms of bolometric magnitudes, as in Robertson's formula, a correction has had to be applied to the measured magnitudes.

In the presence of an evolutionary effect, however, this correction as usually defined ceases to be directly ascertainable by empirical means, since it depends on the evolutionary effects in the spectrum of the source. The process adopted by Sandage of allowing for these effects appears to be not well defined. The interpretation of the observable data can be made quite clear however by the following analysis, by which reference to theoretical bolometric magnitudes is made unnecessary. Our treatment is somewhat analogous in principle to one given by McVittie (**14**) but differs in the important matter of dealing with evolution.

4. *Astronomical distance related to photo- or radio-magnitudes.*—Suppose that from a source whose permanent spatial coordinates are (ρ, θ, ϕ) in the general model of metric (1.3) the energy of radiation which is emitted in the waveband $(\lambda, \lambda + d\lambda)$ in the interval $(t, t + dt)$ is

$$E(\lambda, t)\, d\lambda\, dt.$$

The function $E(\lambda, t)$ will depend on the type of source considered. This radiation will subsequently lie in a spherical shell which will pass over the origin $\rho = 0$ in the interval $(t_0, t_0 + dt_0)$, say. At time t_0 the integrated area of the surface

of this sphere according to fundamental observers on it (total proper area) will be

$$4\pi\rho^2/(1+\rho^2/a^2)^2.$$

The total integrated energy of the radiation apparent to such observers will only be

$$E(\lambda, t)\, d\lambda\, dt\{R(t)/R_0\},$$

due to the degradation caused by the recession of the source (eqn. (2.1)). Accordingly, the flux of energy per unit proper area at the origin will be

$$\frac{E(\lambda, t)\, d\lambda \left\{\frac{dt}{dt_0} \cdot \frac{R(t)}{R_0}\right\}}{4\pi\rho^2/(1+\rho^2/a^2)^2}.$$

Since ρ is constant for the source it follows easily from (1.5) that $dt/dt_0 = R(t)/R_0$, and so the flux of energy becomes

$$\frac{E(\lambda, t)\, d\lambda}{4\pi D^2}, \tag{4.1}$$

where D is given by (3.7); we may regard D as a useful theoretical parameter.

To get the energy actually registered on a photographic plate, or other detective device, in the received waveband $d\lambda_0$ we now require to multiply (4.1) by a function $s(\lambda_0)$ of *received* wave-length to allow for the selective effect of the atmosphere and sensitivity of the plate etc. The energy registered is therefore

$$\frac{\left(\frac{1}{1+\delta}\right) s(\lambda_0) E\left(\frac{\lambda_0}{1+\delta}, t\right) d\lambda_0}{4\pi D^2},$$

where we have put $\lambda = \lambda_0/(1+\delta)$, $d\lambda = d\lambda_0/(1+\delta)$, in accordance with (2.1) and (2.2). Taking account now of radiation emitted over all wave-lengths we find for the registered heterochromatic intensity

$$l^*_{\text{het}} = \frac{\left(\frac{1}{1+\delta}\right)\int_0^\infty s(\lambda_0) E\left(\frac{\lambda_0}{1+\delta}, t\right) d\lambda_0}{4\pi D^2}. \tag{4.2}$$

We note that if we put $s(\lambda_0) = 1$, so that all energy arriving outside the atmosphere is registered, we get the bolometric intensity l^*_{bol} already given by (3.1). This is a consequence of the fact that the total rate of emission $E^*(t)$ may be written

$$\begin{aligned} E^*(t) &= \int_0^\infty E(\lambda, t)\, d\lambda \\ &= \left(\frac{1}{1+\delta}\right)\int_0^\infty E\left(\frac{\lambda_0}{1+\delta}, t\right) d\lambda_0. \end{aligned} \tag{4.3}$$

As a particular application of the formula (4.2) we may write down the expression for the luminosity of a *local* source of the same general type, but now (conventionally) at a distance of 10 parsecs from the observer at epoch t_0:

$$L^*_{0(\text{het})} = \frac{\int_0^\infty s(\lambda_0) E(\lambda_0, t_0)\, d\lambda_0}{4\pi 10^2}. \tag{4.4}$$

Here, putting the parameter t equal to t_0 allows for the distinction between near and distant sources which might arise from an evolutionary effect. Let m_{het} be the registered apparent magnitude of the distant source and $M_{0(\text{het})}$ the absolute registered magnitude of the nearby source, so that

$$m_{\text{het}} - M_{0(\text{het})} = 2{\cdot}5 \log_{10} (L^*_{0(\text{het})}/l^*_{\text{het}}). \tag{4.5}$$

It follows from (3.5) and (4.5) that the relation between the registered and bolometric magnitudes is

$$m_{\text{bol}} - M_{0(\text{bol})} = m_{\text{het}} - M_{0(\text{het})} - K \tag{4.6}$$

where

$$K = 2{\cdot}5 \log_{10} \left\{ \frac{\left(\frac{1}{1+\delta}\right) \int_0^\infty E\left(\frac{\lambda_0}{1+\delta}, t\right) d\lambda_0}{\int_0^\infty E(\lambda_0, t_0)\, d\lambda_0} \right\}$$

$$- 2{\cdot}5 \log_{10} \left\{ \frac{\left(\frac{1}{1+\delta}\right) \int_0^\infty s(\lambda_0) E\left(\frac{\lambda_0}{1+\delta}, t\right) d\lambda_0}{\int_0^\infty s(\lambda_0) E(\lambda_0, t_0)\, d\lambda_0} \right\}$$

$$= 2{\cdot}5 \log_{10} \left\{ \frac{E^*(t)}{E^*(t_0)} \right\}$$

$$- 2{\cdot}5 \log_{10} \left\{ \frac{\left(\frac{1}{1+\delta}\right) \int_0^\infty s(\lambda_0) E\left(\frac{\lambda_0}{1+\delta}, t\right) d\lambda_0}{\int_0^\infty s(\lambda_0) E(\lambda_0, t_0)\, d\lambda_0} \right\}, \tag{4.7}$$

noting (4.3).

The correction to the measured magnitudes which has been made in the past, before substitution in theoretical relations involving bolometric magnitudes, is the K correction as it is defined in our equation (4.6). It is the correction adopted by Sandage, given by his equation B_5 in the appendix to the paper referred to in Section 3. However, Sandage gives an analytic expression for K only for the special case of the absence of evolution. This expression, which in our notation we shall denote by K_0, is got by putting $t = t_0$ in (4.7), so that

$$K_0 = 2{\cdot}5 \log_{10} (1+\delta) - 2{\cdot}5 \log_{10} \left\{ \frac{\int_0^\infty s(\lambda_0) E\left(\frac{\lambda_0}{1+\delta}, t_0\right) d\lambda_0}{\int_0^\infty s(\lambda_0) E(\lambda_0, t_0)\, d\lambda_0} \right\}. \tag{4.8}$$

In order to analyse the red shift data by Robertson's bolometric formula, which is equivalent to a series expansion of our (3.6) and takes account of evolution, Sandage's procedure is to make the K_0 correction, and then to add an estimated correction to make up the total K correction appropriate to evolution. This procedure therefore does not take account of the fact, as shown by (3.6) and (4.7), that part of such an additional correction, viz the term

$$2{\cdot}5 \log_{10} \left\{ \frac{E^*(t)}{E^*(t_0)} \right\},$$

is necessarily cancelled out by the same expression with opposite sign in (3.6). This latter term in Robertson's formula is in fact estimated by Sandage separately. The following procedure appears therefore to be more direct and employs terms that are all well defined.

It may be seen that the only relation which it is necessary to consider is that got from (3.6), (4.6), and (4.7), namely

$$\log_{10} D = 0{\cdot}2\left[m_{\text{het}} - M_{0(\text{het})} + 2{\cdot}5 \log_{10} \left\{ \frac{\left(\frac{1}{1+\delta}\right)\int_0^\infty s(\lambda_0) E\left(\frac{\lambda_0}{1+\delta}, t\right) d\lambda_0}{\int_0^\infty s(\lambda_0) E(\lambda_0, t_0)\, d\lambda_0} \right\} \right] + 1, \tag{4.9}$$

in which no reference to bolometric magnitudes appears. Employing the relation (4.8) for K_0, which may be regarded as the orthodox magnitude correction, this being ascertainable from sources of the required type for which $\delta \simeq 0$, we further simplify (4.9) to the form

$$\log_{10} D = 0{\cdot}2\left[m_{\text{het}} - M_{0(\text{het})} - K_0 + 2{\cdot}5 \log_{10} \left\{ \frac{\int_0^\infty s(\lambda_0) E\left(\frac{\lambda_0}{1+\delta}, t\right) d\lambda_0}{\int_0^\infty s(\lambda_0) E\left(\frac{\lambda_0}{1+\delta}, t_0\right) d\lambda_0} \right\} \right] + 1. \tag{4.10}$$

The first three terms on the right are directly derived from observation and empirical calculation. The fourth term is an unknown quantity which would vanish in the absence of evolutionary trends in the observed spectra. It may be expanded, as shown in Section 5, in powers of the red shift δ. The expression for D, given by (3.7), will also be expanded in powers of δ with coefficients depending on the parameters $\alpha_1, \alpha_2, \ldots$ of the cosmological model (1.3). The equation (4.10) therefore constitutes a direct theoretical relation between two observable quantities—registered apparent magnitude and red shift. Comparison with observation will therefore provide immediate relations between the unknown parameters of the best fitting cosmological model, and also information regarding its evolution, if any.

The result corresponding to our equation (4.10) which is obtained by McVittie in his treatment of the problem mentioned in Section 3 may be regarded as less satisfactory for the following reasons. In his formula (**14**, eqn. 8.614, p. 157) McVittie separates the total 'correction' term into two parts, one that he regards as essentially a red shift correction since it does not vanish even in the absence of evolution, and the other depending on evolution which is termed the Stebbins-Whitford correction. This latter term, however, is misleading, since this part of his correction could be non zero even if the Stebbins-Whitford effect is zero. This is because the observations of Stebbins and Whitford would detect only an evolutionary change in the relative intensities at different wave-lengths and not an evolutionary change in the absolute intensities. More important is the criticism that McVittie's red shift correction contains his evolutionary parameter at the value corresponding to the epoch of *emission*, and is therefore not a term that can be evaluated directly from observations of local spectra, for which we can put $t \simeq t_0$. This term is in fact expanded by McVittie

in powers of δ whose coefficients correspond to the epoch of emission and therefore, in virtue of our equation (2.3), must themselves be regarded as unknown functions of δ.

The advantages of separating out everything that can be evaluated empirically, as in our own analysis, and leaving terms that are non-zero only in the presence of evolutionary effects, need to be emphasized.

5. *The red shift—magnitude relation in series form.*—The theoretical expression for the quantity D which occurs in our formula (4.10) is given by (3.7). Accordingly, using (1.4) and (1.6), D may be expanded in powers of τ, and in turn in terms of δ by (2.3). Thus, after some calculation, we derive for $\log_{10} D$ in terms of δ, as far as the term in δ^2, the expression

$$\log_{10} D = \log_{10}\left(\frac{c\delta}{\alpha_1}\right) + \frac{1{\cdot}086}{5}\left[\left(1+\frac{\alpha_2}{\alpha_1^2}\right)\delta + \left\{-\frac{7}{12} - \frac{5}{6}\frac{\alpha_2}{\alpha_1^2} + \frac{3}{4}\left(\frac{\alpha_2}{\alpha_1^2}\right)^2 - \frac{1}{3}\frac{\alpha_3}{\alpha_1^3} - \frac{4}{3}\frac{c^2}{a^2\alpha_1^2}\right\}\delta^2\right]. \tag{5.1}$$

It is to be noticed that the coefficient of δ^2 involves the third derivative of the metric factor $R(t)$ for the epoch t_0, as well as the curvature of space at this epoch. Thus any attempt at finding the curvature from the red shift—magnitude relation alone is not to be contemplated. Indeed, in view of the present degree of uncertainty in observations, any analysis of the individual contributions to a term in δ^2 is likely to be of little value. Nevertheless if, as is likely, the range of δ to which red shifts are measured can be extended beyond the present limit of $\delta \doteq 0{\cdot}2$ a term in δ^2 might well be formally allowed for in fitting the observational data to the theoretical relation. This would increase the accuracy obtained for the important coefficient of δ. Having remarked this we shall now carry our theoretical expansions to order δ only.

For the fourth term on the right hand side of (4.10), which depends on the epoch t of emission unless evolutionary effects are absent, we write

$$2{\cdot}5\log_{10}\left[\frac{\int_0^\infty s(\lambda_0)E\left(\frac{\lambda_0}{1+\delta}, t_0\right)d\lambda_0 - \tau\int_0^\infty s(\lambda_0)\dot{E}\left(\frac{\lambda_0}{1+\delta}, t_0\right)d\lambda_0 + O(\tau^2)}{\int_0^\infty s(\lambda_0)E\left(\frac{\lambda_0}{1+\delta}, t_0\right)d\lambda_0}\right]$$

$$= 2{\cdot}5\log_{10}\left[1 - \tau\left\{\frac{\int_0^\infty s(\lambda_0)\dot{E}(\lambda_0, t_0)\,d\lambda_0}{\int_0^\infty s(\lambda_0)E(\lambda_0, t_0)\,d\lambda_0}\right\} + O(\tau^2)\right]$$

$$= -1{\cdot}086\left[\left\{\frac{\int_0^\infty s(\lambda_0)\dot{E}(\lambda_0, t_0)\,d\lambda_0}{\alpha_1\int_0^\infty s(\lambda_0)E(\lambda_0, t_0)\,d\lambda_0}\right\}\delta + O(\delta^2)\right]. \tag{5.2}$$

In this we have used the expansion of τ in terms of δ given by (2.3). The notation $\dot{E}(\lambda_0, t_0)$ means

$$\frac{\partial}{\partial t_0}\{E(\lambda_0, t_0)\}_{\lambda_0 \text{ const.}}.$$

A point of procedure must now be raised regarding the K_0 term in (4.10). Since K_0 can be evaluated completely by examination of typical local spectra we might forego the expansion of this term in powers of δ and apply the complete correction to the apparent magnitudes before fitting the data to a linearised red shift—magnitude relation. This was in fact the procedure adopted by Sandage in his analysis of the data of Humason, Mayall and Sandage. Alternatively, we may include in the theoretical red shift—magnitude relation the linear part of the K_0 term only and apply the uncorrected data directly to this relation, allowing for the coefficient of the linear part of K_0 afterwards as a known quantity. If a term in δ^2 were formally allowed for in fitting the data the coefficient of δ^2 would of course be different in the two methods. We shall derive the two formulae for these two methods.

Method (i). In this method the whole of the K_0 correction is made first to each magnitude so that by (4.10), (5.1) and (5.2) the appropriate red shift—magnitude relation to order δ is

$$m_{\text{het}} - K_0 = 5\log_{10}(c\delta) + 1{\cdot}086\left(1 + \frac{\alpha_2}{\alpha_1^2} + \lambda^*\right)\delta + M_{0(\text{het})} - 5\log_{10}\alpha_1 - 5, \quad (5{\cdot}3)$$

where

$$\lambda^* = \frac{\int_0^\infty s(\lambda_0)\dot{E}(\lambda_0, t_0)\,d\lambda_0}{\alpha_1\int_0^\infty s(\lambda_0)E(\lambda_0, t_0)\,d\lambda_0}. \quad (5{\cdot}4)$$

Method (ii). For this method we require the linear part of K_0. It is easily derived from (4.8) that this is

$$1{\cdot}086\left\{\frac{\int_0^\infty s(\lambda_0)\{E(\lambda_0, t_0) + \lambda_0 E'(\lambda_0, t_0)\}\,d\lambda_0}{\int_0^\infty s(\lambda_0)E(\lambda_0, t_0)\,d\lambda_0}\right\}\delta, \quad (5{\cdot}5)$$

where

$$E'(\lambda_0, t_0) \equiv \frac{\partial}{\partial\lambda_0}\{E(\lambda_0, t_0)\}_{t_0\,\text{const.}}$$

Accordingly, the appropriate red shift—magnitude relation is now

$$m_{\text{het}} = 5\log_{10}(c\delta) + 1{\cdot}086\left(1 + \frac{\alpha_2}{\alpha_1^2} + \kappa^* + \lambda^*\right)\delta + M_{0(\text{het})} - 5\log_{10}\alpha_1 - 5, \quad (5{\cdot}6)$$

where

$$\kappa^* = \frac{\int_0^\infty s(\lambda_0)\{E(\lambda_0, t_0) + \lambda_0 E'(\lambda_0, t_0)\}\,d\lambda_0}{\int_0^\infty s(\lambda_0)E(\lambda_0, t_0)\,d\lambda_0}. \quad (5{\cdot}7)$$

6. *Stebbins-Whitford spectral analysis.*—The κ^* term in (5.6) arises directly from the selective effect of red shift and would occur even in the absence of an evolutionary process. The λ^* term would be non zero only in the case of an evolutionary universe. It should be noticed that if $s(\lambda_0)$ is such that only radiation in a narrow waveband is registered on the detective device, e.g. by the use of colour filters, so that $s(\lambda_0) = 0$ outside this waveband, then the expressions for

κ^* and λ^* can be written, with an approximation depending on the width of waveband employed,

$$\kappa^* = \{E(\lambda_0, t_0) + \lambda_0 E'(\lambda_0, t_0)\}/E(\lambda_0, t_0), \tag{6.1}$$

$$\lambda^* = \dot{E}(\lambda_0, t_0)/\{\alpha_1 E(\lambda_0, t_0)\}, \tag{6.2}$$

for the neighbourhood of any given wave-length λ_0. It follows from (6.1) that if λ_0 is chosen so that

$$\left.\begin{aligned} E(\lambda_0, t_0) + \lambda_0 E'(\lambda_0, t_0) &= 0, \\ \text{then} \qquad \kappa^* &= 0. \end{aligned}\right\} \tag{6.3}$$

It is clear that this latter condition is likely to be most nearly satisfied at the red end of the spectrum, since the peak of the light curve occurs at shorter wave-lengths so that $E'(\lambda_0, t_0) < 0$ in this region.

The special form of the expressions for κ^* and λ^* given by (6.1) and (6.2) would be particularly applicable to the multi-colour measurements of the type performed by Stebbins and Whitford (**15**, **16**). It might also find useful application to reception at radio wave-lengths which is usually in narrow wavebands (**17**).

To deal with the important Stebbins-Whitford type of spectral analysis by means of our formula (5.6), suppose that by means of filtering the radiation from a distant source is confined to a narrow waveband for which the effective wave-length of received energy is λ_0. Let the corresponding registered magnitude (heterochromatic) be m, and suppose that the registered absolute magnitude for the same waveband of a nearby source ($\delta \doteq 0$) of the same type is M_0. Then, to order δ, equation (5.6) applies with κ^* and λ^* given for narrow wavebands by (6.1) and (6.2). Let all these quantities, with dashes, refer to another narrow waveband elsewhere in the received spectrum. Then by subtracting the two equations written in accordance with (5.6) we obtain

$$m - m' = M_0 - M_0' + 1{\cdot}086\{(\kappa^* - \kappa^{*\prime}) + (\lambda^* - \lambda^{*\prime})\}\delta. \tag{6.4}$$

Thus, if C is the colour index for these two wavebands in the case of the distant source and C_0 that for the local source, then

$$C = C_0 + 1{\cdot}086\{(\kappa^* - \kappa^{*\prime}) + (\lambda^* - \lambda^{*\prime})\}\delta. \tag{6.5}$$

The effect of the red shift on the colour index C can easily be calculated from a knowledge of κ^* and $\kappa^{*\prime}$ derived from the specimen source near the observer. An additional effect on the colour index, arising from a differential effect with respect to wave-length of a systematic evolutionary change in the spectrum, appears if $\lambda^* - \lambda^{*\prime} \neq 0$. Equation (6.5) determines $\lambda^* - \lambda^{*\prime}$ for different pairs of wavebands. This formula appears to be a new one for Stebbins-Whitford type analysis. It does not depend on the assumption that the source radiates as a black body.

By seven colour photoelectric analysis (**16**) Whitford now finds no significant difference in the spectra of elliptic galaxies at varying distances. However, as was remarked in Section 4, this type of analysis provides only *relative* measures of the energy in different colours. Until we possess a knowledge of the absolute value of λ^* for one colour in such spectra we cannot say that the total energy is not changing with epoch. Nor indeed, until we have such knowledge, can we accurately deduce a value for the cosmological parameter α_2 from the red shift—magnitude relation (5.3), or (5.6). As will be shown in a later paper the value

of λ^* can however be derived from the number count data, or a combination of number counts and red shift—magnitude data.

7. *Determination of the cosmological parameters* α_1 *and* α_2.—The observational data in the form of registered (heterochromatic) apparent magnitudes and corresponding red shift can be fitted to our formula (5.3), or (5.6), and the constant coefficients found by least squares. As remarked earlier it might lead to greater accuracy if a term in δ^2 was formally allowed for in this process, even if we cannot hope to analyse this term profitably.

From the relation (5.3), following Method (i) of Section 5, we should therefore derive values for the constants

$$1{\cdot}086\left(1+\frac{\alpha_2}{\alpha_1^{\,2}}+\lambda^*\right), \tag{7.1}$$

and

$$M_{0(\text{het})}-5\log_{10}\alpha_1-5, \tag{7.2}$$

while from (5.6), following Method (ii), we find the constant

$$1{\cdot}086\left(1+\frac{\alpha_2}{\alpha_1^{\,2}}+\kappa^*+\lambda^*\right) \tag{7.3}$$

together with the value for (7.2). The value of κ^* would be given by computation of the expression (5.7) from detailed analysis of typical local spectra and a knowledge of the absorption and sensitivity function $s(\lambda_0)$. We deal effectively, therefore, with formulae (7.1) and (7.2).

Given the registered absolute magnitude $M_{0(\text{het})}$, in the appropriate range of wave-length, of local specimen sources of the general type being observed, we can arrive from (7.2) at the value of the parameter α_1. Since the first approximation to the luminosity distance—red shift relation (5.1) is seen to be $c\delta=\alpha_1 D$, we observe that α_1 is the Hubble constant of the expanding universe. The value of α_1 is very sensitive to the value assigned to $M_{0(\text{het})}$.

Substitution of the value derived for α_1 into (7.1) would lead to the value of the parameter α_2, if we knew the value of the evolutionary parameter λ^* for the present epoch t_0. By identifying the cosmological metric (1.3), in the neighbourhood of any fundamental observer, with the metric of specal relativity, we see that $\alpha_2(=\ddot{R}_0/R_0)$ is proportional to the rate at which two cosmological sources ($\rho=$ constant) in the neighbourhood are accelerating away from one another. Thus the sign of α_2 determines whether the expansion is speeding up or slowing down at the present epoch.

The expression for λ^* given by (5.4) can be put in terms of the rate of change of absolute (heterochromatic) magnitude of a typical local source at the present epoch, thus:

$$\begin{aligned}\dot{M}_{0(\text{het})}&=-\frac{d}{dt_0}\left[2{\cdot}5\log_{10}\left\{\int_0^\infty s(\lambda_0)E(\lambda_0,t_0)\,d\lambda_0\right\}\right]\\&=-2{\cdot}5\left[\frac{\int_0^\infty s(\lambda_0)\dot{E}(\lambda_0,t_0)\,d\lambda_0}{\int_0^\infty s(\lambda_0)E(\lambda_0,t_0)\,d\lambda_0}\right]\log_{10}e\\&=-2{\cdot}5\alpha_1\lambda^*\log_{10}e.\end{aligned}$$

Whence

$$\lambda^*=-0{\cdot}92\dot{M}_{0(\text{het})}/\alpha_1. \tag{7.4}$$

As mentioned in Section 6 a value for λ^* could be obtained with the aid of the number counts. Meanwhile, the sign of α_2 for an evolutionary universe

could perhaps be deduced if we were willing to assign limits of plausibility to $M_{0(\text{het})}$. This course is complicated, however, by the possibility of systematic selection error. This important matter will be dealt with in the next section.

8. *Observational selection effects.*—In the foregoing analysis of this paper the possible dependence of the total power of a source, viz., $\int_0^\infty E(\lambda, t)\, d\lambda = E^*(t)$, on the epoch t of emission has been attributed to a possibly evolutionary character possessed by the source. However, the equations (5.3), (5.6), could equally well apply to an *apparent* evolutionary effect such as would arise by a selection bias by the observer which was a function of the distance of the sources and in favour of progressively more powerful sources. This is a matter of crucial importance for the assessment of a genuine overall evolution in the universe.

The most obvious case of selection bias that might arise is as follows. If the absolute luminosity distribution of the sources is a wide one for every epoch t, then the greater the distance of the detected source the more likely it is that it belongs to the brighter end of the distribution, because of the limitations of the instruments. There are two cases to be considered.

If the universe is not in a steady state and there is no creation of matter then, in accordance with the usual interpretation, all galaxies were created at approximately the same epoch. If with increasing distance the observer selects sources intrinsically more powerful than the standard source near him, when at the same epoch of emission, then this would violate the assumption that we are following the (backward) evolution of the standard source. Since the standard source provides the constant $M_{0(\text{het})}$ in (5.3), or (5.6), it is clear that this selection effect would have the same influence, via the parameter λ^*, on the coefficient of δ found from the data as a genuine evolution of the standard source. Consequently, if λ^* were merely estimated by ascribing plausible limits to evolution, instead of being deduced independently with the aid of the number counts etc., it would lead to a value for α_2/α_1^2 lower than the true value. This pseudo-evolutionary effect might or might not be comparable with, and in the same sense as, that due to a genuine evolution in the time lapse $\tau(=t_0-t)$ between emission and reception of the radiation, but in any case the real situation would be obscured.

If on the other hand the universe is in a steady state then there are two possible selection effects. There is the effect described above arising from a possibly wide scatter in absolute magnitude of the sources as they are continuously formed. In addition there is the possibility that once formed the sources increase in brightness as they grow older. In this case the observer would select the oldest galaxies at great distances as these would be the brightest. This could lead to a very large apparent evolutionary trend in the universe as a whole.

Hoyle (11) has pointed out that the oldest galaxies in the observable region of a steady-state universe will have an age between 10^{10} and 10^{11} years. In fact it can be shown that the oldest galaxies indicating a red shift δ will have an age

$$T \log_e \left\{ \frac{cT\delta}{d(1+\delta)} \right\},$$

where $T(=1/\alpha_1)$ is the reciprocal of the Hubble constant and d is the average distance between galaxies where δ is small. Taking $d \simeq 10^7$ lt. yrs. and $T \simeq 8 \times 10^9$ yrs. we find, even for $\delta \simeq 0{\cdot}2$, the greatest red shift so far measured, this age is 4×10^{10} yrs. On the other hand the lapse of time, τ, between emission and reception is $T \log_e (1+\delta)$ which at $\delta = 0{\cdot}2$ is only $1{\cdot}5 \times 10^9$ yrs. Thus it is possible

that in a steady-state universe an apparent general evolution might exist much greater than in any genuinely evolving universe. Accordingly, if this were not allowed for by an independent determination of λ^*, a value for α_2/α_1^2 too small, or even negative, would be deduced so that the steady state model, on this test at least, would be inadvertently ruled out.

In the case of the first type of apparent evolution, arising from a scatter in absolute magnitude, a basis for the estimation of this effect in both kinds of universe might be found from a study of the luminosity function of galaxies showing small δ.

We point out that the presence of either type of selection effect could be detected by the use of measurements of angular distance between distant galaxies in conjunction with red shift. It will be shown in a later paper that there is a simple relation between angular distance and red shift for two 'average' galaxies showing this red shift, either in a steady-state universe or in a genuinely evolving universe. For instance, in the case of the steady-state universe, if d is the average distance between galaxies where $\delta \simeq 0$ (i.e. where the galaxies are relatively young), then the angular distance, θ, between two galaxies which are at the same distance d apart, but both showing sensible red shift δ, will be given by

$$\theta = \frac{d}{cT}\left(\frac{1+\delta}{\delta}\right).$$

Consequently, if it is found that the galaxies observed are on the average at a greater angular distance apart, this would mean that the observer is viewing galaxies either older than the average age of the galaxies in his own neighbourhood or at least rarer, and therefore probably brighter, than the galaxies of his neighbourhood.

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Royal Holloway College,
Englefield Green,
Surrey:
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References

(1) H. P. Robertson, *Ap. J.*, **82**, 284, 1935.
(2) A. G. Walker, *Proc. Lond. Math. Soc.* (2), **42**, 90, 1936.
(3) G. C. McVittie, *Proc. Phys. Soc.*, **51**, 529, 1939.
(4) O. Heckmann, *Theorien der Kosmologie*, Berlin: Springer, 1942.
(5) H. P. Robertson, *Publ. Astr. Soc. Pac.*, **67**, 82, 1955.
(6) G. Lemaitre, *Ann. Soc. Sci. Bruxelles*, **47**, A, 49, 1927.
(7) E. T. Whittaker, *Proc. Roy. Soc.* A, **133**, 93, 1931.
(8) W. H. McCrea, *Zs. f. Ap.*, **9**, 290, 1934–35.
(9) R. C. Tolman, *Relativity, Thermodynamics and Cosmology*, Oxford: Clarendon Press, 1934, §§ 157–165.
(10) H. Bondi and T. Gold, *M.N.*, **108**, 252, 1948.
(11) F. Hoyle, *M.N.*, **108**, 372, 1948.
(12) G. C. McVittie, *General Relativity and Cosmology*, London: Chapman and Hall, 1956, Chapters 8, 9.
(13) M. L. Humason, N. U. Mayall and A. R. Sandage, *Astron. J.*, **61**, 97, 1956.
(14) G. C. McVittie, *op. cit.*, pp. 154–157.
(15) J. Stebbins and A. E. Whitford, *Ap. J.*, **108**, 413, 1948.
(16) A. E. Whitford, reported in *Sky and Telescope*, **16**, 222, 1957.
(17) See e.g. M. Ryle and P. Scheuer, *Proc. Roy. Soc.* A, **230**, 448, 1955.

A PROPOSED ASTRONOMICAL TEST OF THE "BALLISTIC" THEORY OF LIGHT EMISSION

Herbert Dingle

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Summary

Ritz's " ballistic " theory of light emission is outlined, and the evidence on which it has been regarded as untenable is re-examined. It is shown that the theory as presented by Ritz is indefinite, and that, when it is amended to a form consistent with the postulate of relativity of motion, it is no longer vulnerable to the earlier criticisms. The question of its admissibility is therefore open. It is pointed out that determinations of the " constant of aberration " from observations of distant nebulae might settle this question, and such determinations are recommended.

It is well known that, according to Einstein's restricted theory of relativity, no material body can move with a greater speed than that of light *in vacuo*, namely, about $3 \cdot 10^{10}$ cm/sec. Recent work in nuclear physics, however, seems to conflict with this*, for, according to the uncertainty principle, the uncertainty in the velocity of a body may approximate to infinity. Independently of this, the internal consistency of the restricted relativity theory seems questionable if the postulate of the constancy of the velocity of light is given its usual interpretation of requiring that all bodies present at the origin of a wave of light, whatever their relative motions, must be regarded as remaining thereafter at the centre of the light-sphere†. These difficulties are removed if the postulate be interpreted merely as requiring that the velocity of light relative to its actual material source shall always be c, and this hypothesis was, in fact, advanced long ago by Ritz‡ and was strongly supported by la Rosa among others: it is usually known as the " ballistic " hypothesis of light emission. Though for a time regarded as a serious rival to Einstein's theory, it was ultimately rejected as being contrary to observation, and has now for many years been held to have been definitely disproved. In view of the new difficulties facing Einstein's theory, however, a revision of the hypothesis and of the grounds for its rejection seems desirable.

Ritz's hypothesis is based on the postulate of relativity of motion, which he expresses in the following way§:— " La seule conclusion qui . . . me semble possible, c'est que *l'éther n'existe pas, ou plus exactement, qu'il faut renoncer à se servir de cette image; que le mouvement de la lumière est un mouvement relatif comme tous les autres, que les vitesses relatives seules jouent un rôle dans les lois de la nature.*"

On this basis he makes the following statements concerning the motion of light‖: " les particules lumineuses expulsées en tous sens à l'instant *t se meuvent avec une vitesse radiale constante et remplissent constamment une sphère dont le*

* See, for example, W. Heisenberg, *The Physicist's Conception of Nature*, p. 48, London, 1958

† *Bull. Inst. Phys.* **9**, 314, 1958.

‡ W. Ritz, *Ann. de Chim. et de Phys.*, **13**, 145–275, 1908.

§ *Ibid.*, p. 207.

‖ *Ibid.*, pp. 210–1.

centre est animé du mouvement de translation w qu'avait P à l'instant de l'émission; si w est constant, ce centre continuera donc de coïncider avec P . . . . La vitesse de la lumière dépend donc de celle que possède le corps qui l'émet au moment de l'émission; à partir de cet instant, la vitesse des particules reste invariable, quel que soit le mouvement ultérieur de *P*."

(It should be remarked that, although Ritz speaks of " light particles ", his theory on this point is purely kinematical and implies nothing whatever concerning the nature of light. As Whittaker has pointed out*, a decision on the " ballistic " hypothesis is " without significance one way or the other in the dispute between the wave and corpuscular theories of light ".)

Unfortunately, this statement of the hypothesis is indefinite, since Ritz fails to make clear to what standard of rest the velocities are referred. The ether having been abjured, the phrase, " *mouvement de translation w qu'avait P à l'instant de l'émission* ", leaves us guessing with respect to what *P* (the source of light) has the velocity *w*; and the phrase, " la vitesse des particules reste invariable, quel que soit le mouvement ultérieur de *P* ", seems to indicate that, whatever the standard may be, *P* may have a varying velocity with respect to it, and therefore with respect to the light particles after the moment of their emission. De Sitter, who has given what is generally considered the most convincing disproof of the " ballistic " hypothesis†, tacitly assumes the standard of rest to be the Earth (or perhaps the Sun). He considers observations of a binary star—for simplicity let us suppose the plane of the orbit to pass through the Earth and one component only to be visible. If the orbital velocity is v, the light issuing while the star is approaching the Earth will have velocity $c+v$, and that issuing while it is receding from the Earth will have velocity $c-v$. If D is the distance of the star from the Earth, the former light will take time $D/(c+v)$, and the latter time $D/(c-v)$, to reach the Earth. The orbit calculated from the observations will therefore be very different from the actual orbit, and if the latter is Keplerian the former will not be so, but, as de Sitter showed, may in a particular case imply that the star has three different velocities at the same time. The actual observations, however, do correspond to a Keplerian orbit. We must therefore, said de Sitter, reject the supposition that the velocity of the light depends on that of its source.

If we are entitled to suppose that a member of the Solar System is acceptable as a unique standard of rest, then de Sitter's evidence against the " ballistic " hypothesis is final. But a different interpretation of Ritz's theory was given by Tolman‡, according to whom the theory requires that " throughout its whole path light retains the component of velocity v which it obtained from the original moving source. Thus all the phenomena of optics would occur as though light were propagated by an ether which is stationary with respect to the original source." On this view, therefore, the light particles (or wave-front), at a time t after emission, would be at a distance ct from the source, no matter how the source might have moved, between emission and the instant t, with respect to any standard at all.

Ritz regrettably died shortly after his paper appeared, and he therefore had no opportunity of elucidating his meaning. It is clear, however, that the

* E. T. Whittaker, *History of the Theories of Aether and Electricity*, Vol. II, p. 39, London, 1953.
† W. de Sitter, *Proc. Amsterdam Acad.*, **15**, 1297, 1913; *B.A.N.*, No. 64, **2**, 163, 1924.
‡ R. C. Tolman, *Phys. Rev.*, **35**, 136, 1912.

only "ballistic" hypothesis that is consistent with the relativity of motion, the absence of a luminiferous ether, and any plausible form of postulate of constant light velocity, is that contemplated by Tolman. Suppose that a body, alone in space, emits a pulse of light. If there is any meaning at all in calling the velocity of light constant, the pulse must continue to move at velocity c *with respect to the source.* In the absence of a universal stationary medium, there is nothing else with respect to which to express its velocity. It will make no difference to the relative velocity of the light and its source if we place a body some light-years away with respect to which the source moves to and fro; so if, from the point of view of that body, the velocity of the source fluctuates between $+v$ and $-v$, the velocity of its light must fluctuate between $c+v$ and $c-v$, and a beam issuing t seconds after an earlier one will reach a distance D from the source precisely t seconds after that earlier one.

If, then, we interpret the "ballistic" hypothesis in this way, clearly the observations cited by de Sitter afford no evidence against it. With respect to the Earth, each particular light-pulse issuing from the star fluctuates in velocity during its journey, in unison with the fluctuations in velocity of the star, and the observations therefore give precisely the same relation of orbital velocity to time as that characterizing the actual orbit. Such fluctuations seem strange only if the idea of a universal ether remains unrecognized in our minds. If we consider only the Earth and the star, the apparent movement of the star will be indistinguishable in character from that which we attribute to the effect of aberration of light. It is only when we take the rest of the universe into account that we realize the convenience of ascribing the first to the orbital motion of the star and the second to the orbital motion of the Earth. It is tempting, of course, to think that the overwhelming measure of this convenience must make the whole material system of the universe a preferred standard of rest, but however that may be in general mechanics (it is unnecessary to consider that question here), it is an impossible supposition in this case, for we are concerned only with the velocity of light in space. If we suppose that the "universe" provides an absolute reference frame for that, then the Michelson–Morley experiment should have shown the Earth's velocity with respect to this frame, but it did not. Accordingly, "universe" becomes in this connection merely another name for "luminiferous ether", and the inapprehensibility of that leaves us with the source of light as the only possible reference body with respect to which the velocity of light can be postulated to be constant.

It should be unnecessary to add that, on the "ballistic" theory, the simple addition law of composition of velocities holds, so that the velocity of light with respect to the *receiving* body may exceed c.

Tolman, however, went on to propose a different test of Ritz's hypothesis. He held that the hypothesis would require a positive result in the Michelson–Morley experiment if the light used came from an astronomical source moving with respect to the Earth, for in that case the velocities of the light along the two arms of the apparatus would be different, and a change of fringe system would be expected on rotating the apparatus. Later, the experiment was repeated with celestial light by Tomaschek*, and independently by Miller†,

* R. Tomaschek, *Ann. d. Phys.*, **73**, 105, 1924.
† D. C. Miller, *Proc. Nat. Acad. Sci.*, **11**, 306, 1925.

but with the same result as in the original performance. An experiment by Majorana* with moving terrestrial light also gave a null result. Tolman accordingly concluded that the "ballistic" hypothesis was disproved.

It appears, however, on general grounds that experiments of this kind can afford no evidence on the matter, for, since the only velocity concerned is the relative velocity of the apparatus and the incident light, an experiment which failed to reveal it for one value could hardly do so for another. The equality of the two paths taken by the light in the Michelson–Morley experiment is independent of the velocity of the light when it enters the apparatus. If the velocity should change, the time taken by the light must change equally along both paths, and no differential effect would be expected to be observed. In terms of interference fringes, it should be noted that on the "ballistic" theory the frequency of the (monochromatic) light received is proportional to the velocity of its reception. A change in that velocity would therefore show itself as a change in frequency, and in no other way. But a similar change of frequency would occur with a change in velocity of the radiating body on any theory: it would be observed as a Doppler effect. Consequently, any change in the fringe system arising from a change in the velocity of the radiating body relative to the apparatus would not distinguish between the "ballistic" theory and Einstein's theory of invariability of velocity of light *reception*.

In view of these general considerations, it is necessary to examine Tolman's argument to see where the discrepancy lies. He supposes the experiment performed with light from the Sun when that body is moving along the direction of the arm AB with velocity v with respect to the apparatus, and he writes:—"It is easy to see that the Ritz theory would lead us to expect $c+v$ for the velocity of light in the direction AB, $c-v$ for the velocity in the opposite direction." But the theory leads us to expect nothing of the kind. The velocity of the light in the direction AB is certainly supposed to be $c+v$ with respect to the apparatus, but the velocity in the opposite direction depends on the law of reflection of light. On this the Ritz theory says nothing, and the only reasonable assumption is that on this, as on all non-ether theories, light is reflected as a perfectly elastic body, and leaves the mirror at the same velocity with respect to it as that which it had on incidence (but of course with opposite sign). In that case the shift of fringe system assumed by Tolman would not be expected to occur.

This appears to exhaust the evidence on which the "ballistic" theory has been rejected, and we must conclude from these considerations that at present there is no basis at all on which to decide for or against the theory as interpreted by Tolman, namely, that the velocity of light proceeding from a material source remains constant, at a value c, with respect to that source, and is received at velocity $c+v$ (vectorial addition) by a body moving at velocity v with respect to the source at the time of reception. The purpose of this note is to suggest how legitimate evidence might be obtained.

The "constant of aberration" is, to a first approximation, proportional to the ratio, v/V, of the orbital velocity of the Earth to the velocity of the light received from the celestial body concerned, both measured with respect to the Sun as a standard of rest. On the "ballistic" theory this quantity should

* Q. Majorana, *Phil. Mag.*, **37**, 145, 1919.

vary with the radial velocity of the celestial source, while on the view that light is always received at velocity c, it should be constant. Stellar velocities are too small to distinguish between these possibilities, but a distant nebula might move fast enough. The light from a nebula with a velocity of recession of 30 000 km/sec, for instance, should, on the "ballistic" theory, be received at a velocity $0{\cdot}9c$, and the aberrational constant should accordingly be about 23″ instead of 20″·5—an easily detectable difference. Still larger differences are within range of detection.

Unfortunately, if such a nebula yielded the normal value for the constant the result would not necessarily be conclusive, owing to the uncertainty which still remains as to whether the nebular red-shift is properly attributed to recessional movement. On the other hand, if the larger value were obtained it would be difficult to think of any possible explanation except that the red-shift is due to recession and the "ballistic" theory is correct. Furthermore, a method would become available of finding the recessional velocity of any nebula bright enough to be directly photographed. These possibilities make it very desirable that this observation should be made.

104 *Downs Court Road,*
Purley, Surrey:
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NOTE ON THE MEASUREMENTS OF RADIO EMISSION FROM CASSIOPEIA A

V. Pletchkov and V. Razin

Measurements of radio emission from the discrete source Cassiopeia A at a frequency of 300 Mc/s have been attributed to us by a number of authors (**1, 2, 3**). We wish to point out that this is erroneous. We have made the measurements of intensities of radio emission from the Cassiopeia A and two more discrete sources at wave-lengths of 9·7 cm (frequency is 3100 Mc/s) (**4**) and 3·2 cm (**4, 5**) only. It should be noted that the flux density of radio emission from Cassiopeia A cited in the first three references for a frequency of 300 Mc/s is very close to that measured by one of us (Razin) at a frequency of 3100 Mc/s, viz. $1{\cdot}3 \times 10^{-23}$ w/m^2 (c/s) (**4**).

References

(1) C. L. Seeger, Symp. No. 4 of the IAU on Radio Astronomy, 156, 1957.
(2) N. G. Roman and F. T. Haddock, *Ap. J.*, **124**, 35, 1956.
(3) G. R. Whitfield, *M.N.*, **117**, 680, 1957.
(4) V. M. Pletchkov and V. A. Razin, Publications of the 5th Cosmogonical Conference, Moscow, 1956, p. 430.
(5) V. A. Razin and V. M. Pletchkov, Symp. No. 4 of the IAU on Radio Astronomy, 155, 1957.

РЕЗЮМЕ ДОКЛАДОВ, В ПЕРЕВОДЕ НА РУССКИЙ ЯЗЫК

О ФОРМЕ КОМЕТНЫХ ХВОСТОВ

Б. Л. Мик

На основе представления о комете, как о рое частиц образовавшихся внутри кометы вблизи перигелия, вычислено пространственное распределение частиц разных размеров для ряда позднейших моментов. Отделение некоторых частиц от главной части кометы обусловлено световым давлением (или динамически эквивалентными эффектами) и может быть представлено как ослабление солнечного притяжения, включая, конечно, даже перемену знака. Обсуждаются некоторые свойства геометрического места частиц (с разными размерами и разным эффективным притяжением), одновременно покидающих комету; некоторые из полученных результатов применяются к частному случаю кометы Аренда-Ролана. Теория дает прямое объяснение видимого острого выступа, направленного к Солнцу, который наблюдался у этой кометы, как результата веерообразного распределения отделившихся частиц, которое может наблюдаться только если рассматривать его из пунктов лежащих в его плоскости или очень близко к ней.

УШИРЕНИЕ ВСЛЕДСТВИЕ СТОЛКНОВЕНИЙ И СДВИГ РЕЗОНАНСНОЙ ЛИНИИ КАЛЬЦИЯ

В. Р. Хайндмарш

Измерены сдвиг вследствие столкновений и уширение линий Ca 4227 Å, обусловленные внешним давлением гелия. Линия получалась в поглощении в результате прохождений белого света через пары кальция в присутствии гелия при различных давлениях меньше 1 атм. Спектральное разрешение было обеспечено решеткой Бабкока, использованной в новом Оксфордском спектрогелиографе. Полуширина линии, обусловленная затуханием, была найдена равной $1{,}72 \pm 0{,}12 \cdot 10^{-20}$ см$^{-1}$ на 1 атом на 1 см^3 гелия, а сдвиг в фиолетовую сторону составлял $0{,}05 \pm 0{,}04 \cdot 10^{-20}$ см$^{-1}$ на 1 атом на 1 см^3. Поэтому отношение уширения к сдвигу $\geqslant 19$, что значительно больше величины 2,76, предсказываемой теорией Линдхольма. Сдвиг также, согласно теории, должен быть к красному концу.

Обсуждаются возможные объяснения этого расхождения. Показано, что отталкивательные силы близкодействия, действующие между атомами кальция и гелия должны учитываться так же, как и силы дальнодействия ван дер Ваальса. Из измерений на основе теории Линдхольма оценены коэффициенты для сил обоих типов. В заключение показано, что в условиях Солнца коэффициент затухания для Ca 4227 Å меньше $2{,}2 \cdot 10^{-3}$ см^3 для $\tau = 0{,}1$ и что сдвиг, обусловленный столкновениями, дает пренебрежимо малый вклад в красное смещение солнечных линий.

СДВИГИ ЛИНИЙ В СПЕКТРЕ АРГОНА ВСЛЕДСТВИЕ СТОЛКНОВЕНИЙ

В. Р. Хайндмарш и К. А. Томас

Были измерены обусловленные столкновениями сдвиги двух линий аргона λ 7067 Å и λ 6965 Å; линии возбуждались постоянным током при искровом разряде в чистом аргоне. Линии изучались при помощи интерферометра Фабри-Перо, скрещенного с призменным спектрографом Хильгера. Диаметры интерференционных колец измерялись для ряда давлений аргона и токов разряда. Показано, что измеренные сдвиги были обусловлены столкновениями между двумя атомами аргона, а не влиянием заряженных частиц. Сдвиг для λ 7067 составляет: $-0{,}61 \pm 0{,}05 \cdot 10^{-20}$ см$^{-1}$ на 1 атом на 1 см^3 аргона, и для λ 6965: $-0{,}65 \pm 0{,}05 \cdot 10^{-20}$ см$^{-1}$ на 1 атом на 1 см^3 аргона; минус соответствует смещению в красный конец спектра. Эти результаты наводят на мысль, что силы близкодействия отталкивания между атомами аргона могут быть мало существенны при сдвигах линий, в отличие от случая линии кальция λ 4227 Å, возмущенной гелием.

ИЗМЕРЕНИЯ УГЛОВОГО ДИАМЕТРА ИСТОЧНИКОВ РАДИОИЗЛУЧЕНИЯ ЛЕБЕДЬ (19N4A) И КАССИОПЕЯ (23N5A) НА ВОЛНЕ 10,7 СМ

Б. Роусон

Описывается аппаратура, использованная для измерения угловых диаметров источников радиоизлучения на волне 10,7 см. Приведены результаты измерений для двух взаимно-перпендикулярных направлений как для Кассиопеи (23N5A), так и для Лебедя (19N4A). Эти результаты показывают, что первый источник характеризуется круговой симметрией и имеет примерно тот же размер, что и на метровых волнах, в то время как второй источник заметно асимметричен с большой осью, которая, возможно, несколько больше на более высоких частотах. Позиционный угол большой оси составляет $109° \pm 2°$.

ЭЛЛИПСОИД СКОРОСТЕЙ И ГРАВИТАЦИОННЫЙ ПОТЕНЦИАЛ ГАЛАКТИКИ

П. А. Уэйман

В 1956 г. М. Шмидт построил модель гравитационного потенциала Галактики на основании обработки круговых скоростей, полученных из 21-см радионаблюдений и других данных. Выясняется, что с небольшими изменениями модель Шмидта можно рассматривать как принадлежащую к типу стационарных галактик, описанному в 1941 г. Каммом. Эллипсоид скоростей в галактиках этого типа может быть трехосным и можно исследовать изменение этих осей в различных частях Галактики. В качестве одного из результатов получается наклон „ линии вертексов " к галактической плоскости на сравнительно небольшом удалении от этой плоскости.

РАДИОИЗЛУЧЕНИЕ ИЗ РАЙОНА „ СВЕРХГАЛАКТИКИ "

Дж. Р. Шейкшафт и Дж. Э. Болдуин

Наблюдения проводились при помощи кембриджского радиотелескопа на волне 1,9 м. На основе этих наблюдений были построены изофоты области неба 10^h–14^h с целью исследования ряда деталей, которые были приписаны излучению от „ местной сверхгалактики ". Корреляция распределения эмиссии с данными подсчета галактик оказалась не достаточно тесной. Наблюдаемые интенсивности в несколько раз больше, чем можно было ожидать для нормальных галактик, а подсчет радиоисточников показал, что этот результат нельзя объяснить малой долей галактик, имеющих аномально высокие светимости в радиодиапазоне. Радиояркость одной из деталей значительно больше, чем яркость всего фона внегалактического излучения. Высказывается предположение, что эти детали в действительности не связаны со „ сверхгалактикой ".

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Сделана переоценка опубликованной ранее теоретической работы по соотношению „ красное смещение—звездная величина ". Выведены новые формулы, которые специально предназначены для практического решения космологической проблемы и обнаружения эволюционных эффектов.

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